MATHEMATICAL MODELING OF AN RLC CIRCUIT AND ANALYSING THE RESPONSES

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ABSTRACT: This paper explains basic concepts in the field of signal processing. Differential equations are very important to modern science and engineering. Firstly, an RLC circuit is modeled mathematically by ordinary differential equation. The responses of the series RLC circuit are analyzed for three different values of R(resistance) corresponding to under damped, critical damped and over damped cases. That models the capacitance, inductance, resistance and driving voltages as functions of time. Modeling electric circuits mathematically gives understanding the subject for all students, not only for electrical engineers but also computer scientists. Finally, the general solution of the ordinary differential equation is obtained.

KEYWORDS: *current, Kirchhoff's voltage law, Ohm's Law, responses, RLC-circuit.*

1. INTRODUCTION

An electrical circuit which consists of a resistor (R), an inductor (L), and capacitor (C), connected in series or in parallel is an RLC circuit. They can be modeled by a second-order ordinary differential equation according to their configurations, in order to analyze each component through time. We need to know some basic concepts to model resistance, capacitance and inductance. Which are (i) voltage is defined as a measure of the potential energy between different points, (ii) current is the rate of electrons travels through a circuit with respect to time, (iii) a resistance is the measure of opposition of passage of current, (iv) capacitance is a measure of the amount of charges that can be stored in a capacitor. When the flow of the current changes how much voltage is induced into a capacitor is measured by inductance According to Kirchhoff's voltage law, an ordinary differential equation in relation to voltage for a parallel RLC circuit is obtained [2].

2. FUNDAMENTAL EQUATIONS

In this section, we will describe the basic equations to derive the equation for our specific RLC Circuit. Each different equation is needed to solve for the voltage to know the values for each different circuit element. Since resistor is a circuit component that resists the flow of electric charge, resistors obey Ohm's law,

$$V = IR,$$

where V is the voltage applied to the resistor, I the circuit current and R(constant) the resistance.

Because of a capacitor is a circuit component that stores a supply of electric charge it obey the equation,

$$V = \frac{Q}{C},$$

where Q is the charge stored in the capacitor, C(constant) the capacitance of the capacitor, and V the resulting voltage. From the above two equations, we get

$$IR = \frac{Q}{C}.$$

Furthermore, the current I at which charge flows through the resistor is equal to the rate at which charge flows out of the capacitor,

so, $$I = -\frac{dQ}{dt}.$$

These two equations give the differential equation,

$$(-\frac{dQ}{dt})R = \frac{Q}{C},$$

$$\frac{dQ}{dt} = -\frac{1}{RC}Q.$$

Finally, the relationship for inductance is

$$V(t) = L\frac{dI}{dt}.$$

Now we can properly derive the equation expected [1].

3. MODELING THE RLC CIRCUIT

Now we can find a model for the circuit we have all basic relationships for the elements in the circuit. Kirchhoff's Voltage law is defined as the sum of the voltage drop across the resistor V_r, the voltage drop across the inductor V_L and the voltage drop across the capacitor V_c is equal to the time varying voltage V_t. By the Kirchhoff's law, we get:

$$V_r + V_L + V_C = V(t). \tag{1}$$

To find the voltage drop across the resistor, we modify the resistance equation for a time varying voltage:

$$RI(t) = V(t),$$

where I(t) is a time varying current.

To find the voltage drop across the capacitor, we should examine the actual component physically. Although the current through a capacitor is the rate of charge passing through it with respect to time electrons do not pass through a capacitor, but instead electrons stick to the negative plate of the capacitor for each electron that leaves the positive plate. The charge on plate is equal and opposite to the charge on the other plate of the capacitor. Therefore, the charge on each plate is the integral of the time varying current function. The equation is

$$\frac{1}{C}\int_0^t I(\tau)d\tau + V(0) = V(t),$$

where C is the capacitance, I(t) the time varying current and V(0) an initial condition of the voltage when the system is turned on [4].

Lastly, the time varying voltage function is considered. This function will be a sine or cosine given the nature of radio waves and will also be a function of time. Thus, we take $V(t) = V_0 \sin \omega t$.

Substituting into equation (1) we get the following equation:

$$L\frac{dI}{dt} + RI(t) + \frac{1}{C}\int_0^t I(\tau)d\tau + V(0) = V_0 \sin \omega t. \qquad (2)$$

Differentiating equation (2) with respect to t yields

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I(t) = V_0\omega\cos\omega t. \qquad (3)$$
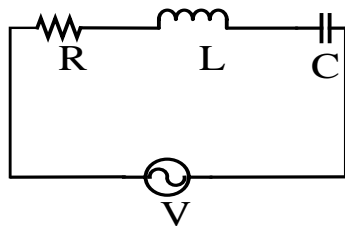

Fig 1. Diagram of a series RLC circuit

4. THE GENERAL SOLUTION OF THE EQUATION

In this section, we will find the general solution of the second order differential equation by finding the homogeneous solution at first and then the particular solution by using the complex exponential waveform.

4.1. Finding the homogeneous solution

To find the homogeneous solution of equation (3), simply set it equal to zero. That is,

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I(t) = 0. \qquad (4)$$

In order to clearer the paper will use prime notation rather than Leibniz notation, gives

$$LI'' + RI' + \frac{1}{C}I = 0,$$

$$I'' + \frac{R}{L}I' + \frac{1}{LC}I = 0. \qquad (5)$$

Taking $I = e^{\lambda t}$, gives the characteristic equation

$$\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0.$$

Solving this equation for λ by the quadratic formula we get

$$\lambda = \frac{-\dfrac{R}{L} \pm \sqrt{\left(\dfrac{R}{L}\right)^2 - \dfrac{4}{LC}}}{2},$$

Simplifying the equation gives

$$\lambda = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}. \qquad (6)$$

By defining called damping rate

$$\alpha = \frac{R}{2L}$$

and natural frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

we get $\lambda_1 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$.

Thus, the roots of the characteristic equation are

$$\lambda_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2},$$
$$\lambda_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2},$$

and the homogeneous solution becomes

$$I_h = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}.$$

The parameters c_1 and c_2 are constants and can be determined by the application of the initial conditions of the system

$$I(t = 0) \quad \text{and} \quad I'(t = 0).$$

The value of the term $\sqrt{\alpha^2 - \omega_0^2}$ determines the behaviour of the response. Three types of responses are possible:

i.   If $\alpha = \omega_0$ then λ_1 and λ_2 are equal and real numbers: no oscillatory behaviour. **(Critically Damped System)**

ii. If $\alpha > \omega_0$ then λ_1 and λ_2 are real numbers but are unequal: no oscillatory behaviour. **(Over Damped System)**

iii. If $\alpha < \omega_0$ then $\sqrt{\alpha^2 - \omega_0^2} = j\sqrt{\omega_0^2 - \alpha^2}$. In this case λ_1 and λ_2 are complex numbers:
$$\lambda_1 = -\alpha + j\sqrt{\omega_0^2 - \alpha^2}, \ \lambda_2 = -\alpha - j\sqrt{\omega_0^2 - \alpha^2}.$$

System exhibits oscillatory behaviour. ($j = \sqrt{-1}$) **(Under Damped System)**

Figure 2 shows the response of the series RLC circuit for three different values of R corresponding to the under damped, critically damped and over damped cases.

A typical critically damped response is shown in Figure 2(a), where it is assumed that $I_n(0) = 0$. This plot is created with the following Matlab code:

```
t=0:.01:6;
it=420.*t.*(exp(-2.40.*t));
plot(t,it);
xlabel('t');
ylabel('(I)');
title('Critically Damped Response for 420t(exp(-2.45t))').
```

Critically Damped Response for 420t(exp(-2.45t))

Under Damped Response for 210 sqrt(2)(exp(-.5t))sin(sqrt(2)t)
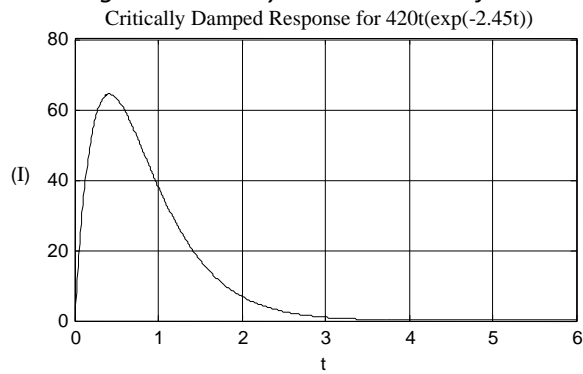
(a) Critical damped response

A typical over damped response is shown in Figure 2(b) where it is assumed that $I_n(0) = 0$. This plot is created with the following Matlab code:

```
t=0:.01:6;

it=8.4.*(exp(-t)-exp(-6.*t));

plot(t,it);

xlabel('t');

ylabel('(I)');

title('Over Damped Response for 8.4(exp(-t)-exp(-6t)')
```

Over Damped Response for 8.4(exp(-t)-exp(-6t)
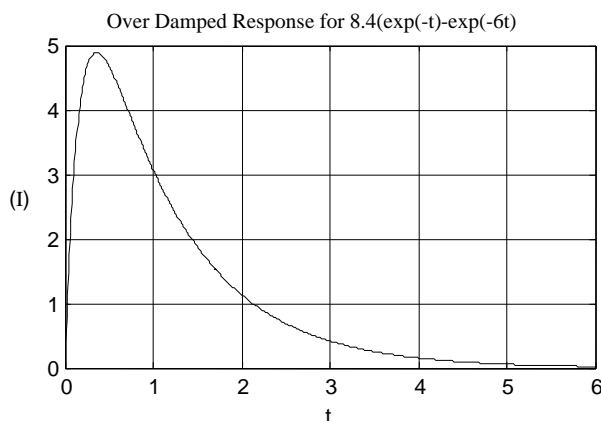
(b) Over damped response

A typical under damped response is shown in Figure 2(c) where it is assumed that $I_n(0) = 0$. This plot is created with the following Matlab code:

```
t=0:.01:10;

it=210.*sqrt(2).*(exp(-.5.*t)).*sin(sqrt(2).*t);

plot(t,it);

xlabel('t');

ylabel('(I)');

title('Under Damped Response for 210 sqrt(2)(exp(-.5t))sin(sqrt(2)t)')
```
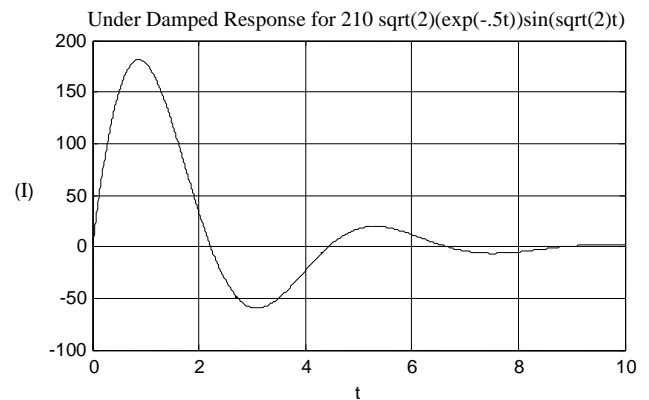
(c) Under damped response
Fig 2. The graphs of typical responses

4.2. Finding the particular solution

The particular solution for the system is found by using the method of undetermined coefficients. The complex exponential waveform is used extensively to simplify analysis. It is possible to use a complex exponential function to express the voltage and current waveforms such as $e^{j\omega t}$, where j is the imaginary number $\sqrt{-1}$. The function is still a sinusoid due to e^{jt} = sint + jcost. The complex exponential waveform is used in place of the standard periodic function since it makes analysis much simpler and integrating or differentiating the function only changes the coefficients. In equation (3) $V_0\omega\sin(\omega t)$ is replaced with $V_0 j\omega e^{j\omega t}$ such that

$$LI'' + RI' + \frac{1}{C} I = V_0 j\omega e^{j\omega t}.$$

Therefore the particular solution is the form

$$I_p = a\, e^{j\omega t},$$

where a is constant to be determined. The first and second derivatives for the particular solution I_p are

$$I_p' = a\omega j e^{j\omega t}\quad, \tag{7}$$
$$I_p'' = -a\omega^2 e^{j\omega t}. \tag{8}$$

Substituting into the original equation gives

$$-La\omega^2 e^{j\omega t}I'' + aR\omega j e^{j\omega t} + \frac{1}{C} a e^{j\omega t} = V_0 j\omega e^{j\omega t}.$$

By dividing the whole equation by $e^{j\omega t}$, we get

$$-a\omega^2 L + Ra\omega j + \frac{1}{C} a = V_0 j\omega.$$

Now isolate V_0. After some simplification, we get

$$a\left(R + j(\omega L - \frac{1}{\omega C})\right) = V_0.$$

Solving for a gives

$$a = \frac{V_0}{R + j(\omega L - \frac{1}{\omega C})}.$$

So the particular solution is then

$$I_p = \frac{V_0}{R + j(\omega L - \frac{1}{\omega C})} e^{j\omega t}. \tag{9}$$

The denominator in equation (9) is the sum of the complex impedances it is the resistance to the current

given a time-varying complex driving function. It is now possible to find the steady-state response for the circuit with the particular solution for the current, after the voltage is applied. When the voltage is first applied to the system, there will be a transient before the system settles into the steady-state response. Equation (9) represents the steady state output current of an RLC circuit, which models the long-term behaviour of the system [3].

The general solution now becomes

$$I(t) = I_h + I_p$$

$$I(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \frac{V_0}{R + j(\omega L - \frac{1}{\omega C})} e^{j\omega t}.$$

4.3. Tuning the RLC circuit

In order to tune the circuit to a certain frequency it is required to attenuate the amplitudes of all other frequencies, and increase the amplitude of the signal with the specified frequency. Then it is assumed that by maximizing the amplitude of the output current at a certain frequency, we have successfully tuned the circuit. To find the amplitude of the output current simply take the magnitude of the equation (9).

$$\left| I_p \right| = \frac{\left| V_0 e^{j\omega t} \right|}{\left| R + j(\omega L - 1/\omega C) \right|}.$$

$$\left| I_p \right| = \frac{V_0}{\left(R^2 + (\omega L - 1/\omega C)^2 \right)^{1/2}}.$$

Now, the circuit can be tuned by changing the inductance and capacitance. The maximum amplitude is achieved when C and L satisfy the following equation

$$\omega L - \frac{1}{\omega C} = 0.$$

Solving for the frequency ω in terms of L and C gives

$$\omega^2 = \frac{1}{LC}.$$

The equation $\omega^2 = \frac{1}{LC}$ maximizes the gain, that is the ratio of the input amplitude over the steady-state amplitude.

By maximizing the gain for a certain frequency, the amplitude for that frequency is also amplified effectively "tuning" the circuit to that signal's frequency. However, this model is not accurate given a large number of different signals with different frequencies.

4.4. Example

We will consider the current I(t) in an RLC circuit with R = 11Ω, L = 0.1H, C = 10^{-2}F, which is connected to a source of EMF, E(t) = 110sin(60·2πt) = 110 sin(377t). Assume that current and capacitor charge are 0 when t = 0 [1].

Substituting R, L, C and E'(t) into (3), we obtain

$$0.1I'' + 11I' + 100I = 110 \cdot 377 \cos(377t).$$

Hence the homogeneous ODE is

$$0.1I'' + 11I' + 100I = 0.$$

Its characteristic equation is

$$0.1\lambda^2 + 11\lambda + 100 = 0.$$

The roots are

$$\lambda_1 = -10, \text{ and} \lambda_2 = -100.$$

The corresponding general solution of the homogeneous ODE is

$$I_h(t) = c_1 e^{-10t} + c_2 e^{-100t}.$$

For the particular solution, put

$$I_p = A \cos(377t) + B \sin(377t).$$

By differentiation and substitution, we get

$$A = -2.71 \text{ and } B = 0.796.$$

Hence in our present case, a general solution of the nonhomogeneous ODE (3) is

$$I(t) = c_1 e^{-10t} + c_2 e^{-100t} - 2.71\cos(377t) + 0.796 \sin(377t).$$

Using the initial conditions I(0) = 0 and I'(0) = 0, give

$$c_1 = -0.323 \quad \text{and} \quad c_2 = 3.033.$$

Hence we obtain

$$I(t) = -0.323 e^{-10t} + 3.033 e^{-100t} - 2.71\cos(377t) + 0.796 \sin(377t).$$
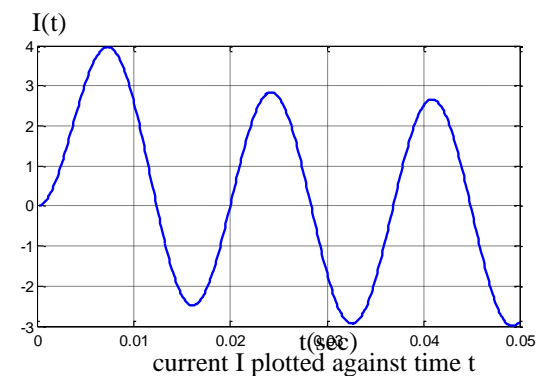

Fig 3. The current I plotted against time t

Figure 3 shows I(t) which practically coincide, except for a very short time near because the exponential terms go to zero very rapidly.

4.5 Example

For the circuit of Figure 4, $I_L(0) = 5$ A, $V_C(0) = 2.5$ V and the 0.5 Ω resistor represents the resistance of the inductor.
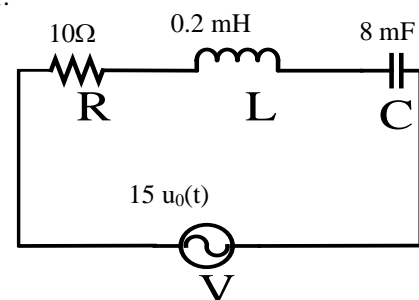

Fig. 4.

The general form of the equation that describes this circuit is

$$RI + L\frac{dI}{dt} + \frac{1}{C}\int_0^t I dt + V_C(0) = 15, t > 0.$$

By differentiating we get the homogeneous differential equation

$$R\frac{dI}{dt} + L\frac{d^2I}{dt^2} + \frac{I}{C} = 0, t > 0$$

or

$$\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{I}{LC} = 0,$$

since the derivatives of the constants $V_C(0)$ and 15 are zero.

By substitution of the known values R, L and C, we get

$$\frac{d^2I}{dt^2} + 500\frac{dI}{dt} + 6000 = 0.$$

The roots of the characteristic equation of this equation are $\lambda_1 = -200$ and $\lambda_2 = -300$.

The total response is the natural response and it is overdamped. Therefore,

$$I(t) = c_1 e^{-200 t} + c_2 e^{-300 t}, \tag{10}$$

where the constants c_1 and c_2 can be determined by using the initial conditions $I(0) = 5A$, $V_C(0) = 2.5$ V.
From the first initial condition, we have

$$I(0) = c_1 + c_2 = 5. \tag{11}$$

Differentiating $I(t) = c_1 e^{-200 t} + c_2 e^{-300 t}$ with respect to t at $t = 0^+$, we get

$$\left.\frac{dI}{dt}\right|_{t=0^+} = -200c_1 - 300c_2.$$

Also, at $t = 0^+$,

$$RI(0^+) + L\left.\frac{dI}{dt}\right|_{t=0^+} + V_C(0) = 15.$$

Solving for $\left.\frac{dI}{dt}\right|_{t=0^+}$ we get

$$\left.\frac{dI}{dt}\right|_{t=0^+} = \frac{15 - 0.5 \cdot 5 - 2.5}{10^{-3}} = 10000.$$

Hence,

$$-200c_1 - 300c_2 = 10000,$$

$$-c_1 - 1.5 c_2 = 50. \tag{12}$$

From equations (11) and (12) we obtain

$$c_1 = 115 \text{ and } c_2 = -110.$$

By substitution into the equation (10), we find the total response as

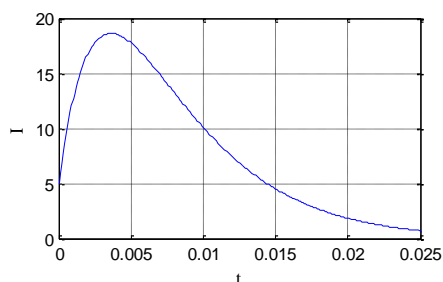$$I(t) = 115 e^{-200 t} - 110 e^{-300 t}.$$

Fig. 5

Matlab code for the total response is
```
t=0:.0001:.025;
I=115*(exp(-200*t))-110*(exp(-300*t));
plot(t,I);
xlabel('t');
ylabel('I');
```
    In this example, differentiation eliminated the right side of the differential equation and thus the total response is just the natural response.

5. CONCLUSION

Many oscillator circuits are applied the RLC circuits such as Radio receivers and television sets for tuning to select a narrow frequency range from ambient radio waves. An RLC circuit is known as a second order circuit since any voltage or current in the circuit can be described by a second-order differential equation for circuit analysis. In this paper, the series RLC circuit's current is modeled to second order differential equation and solved the ordinary differential equation and analyzed the responses of the current. Electricity is a realm that holds up to mathematical models in the micro and macroscopic domains. Although the techniques covered in this paper were very basic, they serve as a springboard for more advanced and elegant analysis.

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Some Applications of Conic Sections in Engineering Field

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ABSTRACT: Conic sections are useful in studying geometry which has wide applications. A conic is a two dimensional figure created by the intersection of a plane and a right circular cone. The four basic types of conic sections are circles, parabolas, ellipses and hyperbolas. In this paper, the standard equations of four types of conic sections are studied. In each type, some basic definitions and standard form of equations with illustrative diagrams are expressed. Then, the applications of conic sections in engineering fields are presented. Firstly, calculating the vertical lengths of the Nagayon Mahabodhi pagoda's arch from our native town, Meiktila by using the parabolic equation is described. Secondly, computing the width of the natural draft cooling tower at the top (in civil engineering) is expressed. Thirdly, calculating the distance to pick up a signal from a small radio transmitter by using the standard form of circle equation is presented. Finally, ellipse equation provides for determining whether a truck could clear or could not clear through the elliptical tunnel.

KEYWORDS: *circle, parabola, ellipse, hyperbola, vertex.*

1. INTRODUCTION

Conic sections were discovered by Menaechmus, Ancient Greece, around 360-350 BC. Menaechmus was a student of Eudoxus, a famous Greek scholar, mathematician at that time. To solve the case of "doubling cube", he concentrated on mean proportions and the use of construction of a cone. Finally, his solution became known as "conic sections". There are four types of conic sections. They are circle, parabola, ellipse and hyperbola. Conic sections can be seen in buildings, factories and arches, etc. Conic sections' equations are very effective to solve the problems of many fields [Larson (2013)].

In this paper, the applications of conic sections in engineering fields are widely presented. These applications are calculating the vertical lengths of the Nagayon Mahabodhi pagoda's arch from our native town, Meiktila and calculating the width of natural draft cooling tower at the top (in civil engineering), computing the distance to pick up signals from a small radio transmitter (in electronics and communication engineering) and determining a truck that can fit or

cannot fit the semielliptical tunnel (in mechanical engineering) [Gamer (1988)].

2. CONIC SECTIONS

A conic section is a curve obtained by the intersection of the surface of a cone with a plane, and has certain features. Depending on the angle of a plane with respect to the cone, a conic section may be a circle, a parabola, an ellipse or a hyperbola [Gamer (1988)].
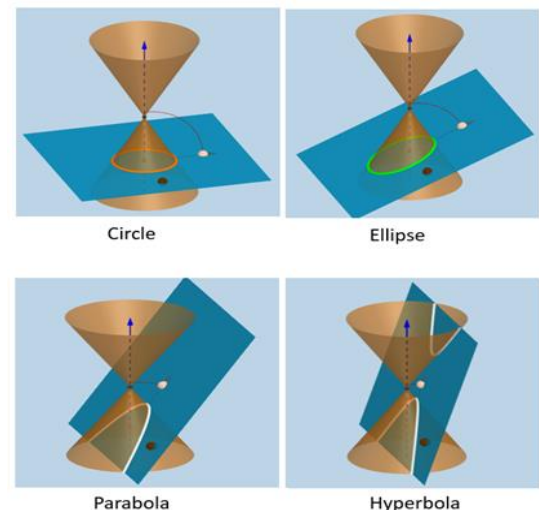

Fig 1 Diagram of Conic Sections

2.1 Circle

A circle is the set of points in a plane whose distance from a given fixed point in the plane is constant [Thomas, Weir, Hass (2009)].

The standard equation for the circle of a radius r centered at the point (h,k) is
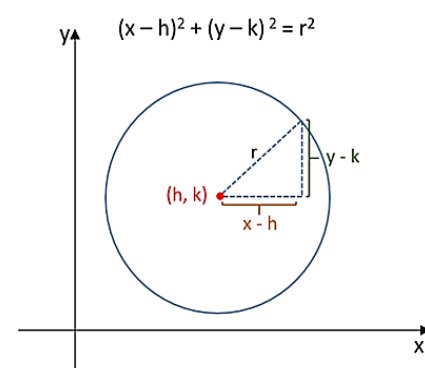
$$(x-h)^2 + (y-k)^2 = r^2. \tag{1}$$

Fig 2 Diagram of Standard Equation for Circle

2.2 Parabola

A parabola is a set of all points (x, y) in a plane that are equidistant from a fixed line called a directrix and a fixed point not on the line called the focus.

The vertex of the parabola is on the line of symmetry half-way between the focus and directrix. The axis of symmetry of the parabola is called the axis of the parabola [Thomas, Weir, Hass (2009)].

Table 1 Standard Equations for Parabolas

Direction	Standard Equations	Vertex	Directrix	Focus
Opens upward	$(x-h)^2 = 4a(y-k)$	(h,k)	$y = k-a$	$(h, k+a)$
Opens downward	$(x-h)^2 = -4a(y-k)$	(h,k)	$y = k+a$	$(h, k-a)$
Opens to the right	$(y-k)^2 = 4a(x-h)$	(h,k)	$x = h-a$	$(h+a, k)$
Opens to the left	$(y-k)^2 = -4a(x-h)$	(h,k)	$x = h+a$	$(h-a, k)$

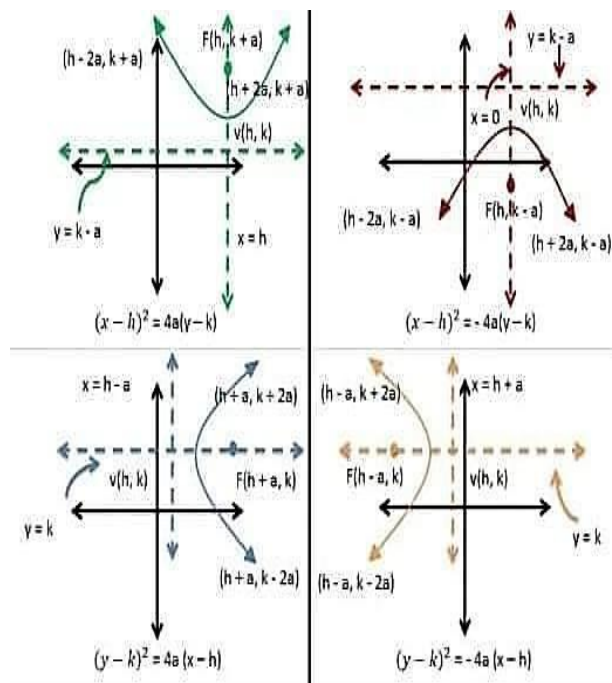a is focus-vertex and vertex-directrix distance.

Fig 3 Diagram of Standard Equations for Parabolas

2.3 Ellipse

An ellipse is the set of points in a plane whose distances from two fixed points in the plane have a constant sum.

Table 2 Standard Equations for Ellipses

Major axis	Standard Equations	Vertices	Foci
Horizontal	$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$	$(h \pm a, k)$	$(h \pm \sqrt{a^2 - b^2}, k)$
Vertical	$\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$	$(h, k \pm a)$	$(h, k \pm \sqrt{a^2 - b^2})$

In each case, a is always greater than b since a is the semimajor axis and b is the semiminor axis [Barannan, Esplen, Gray (2003)].

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$
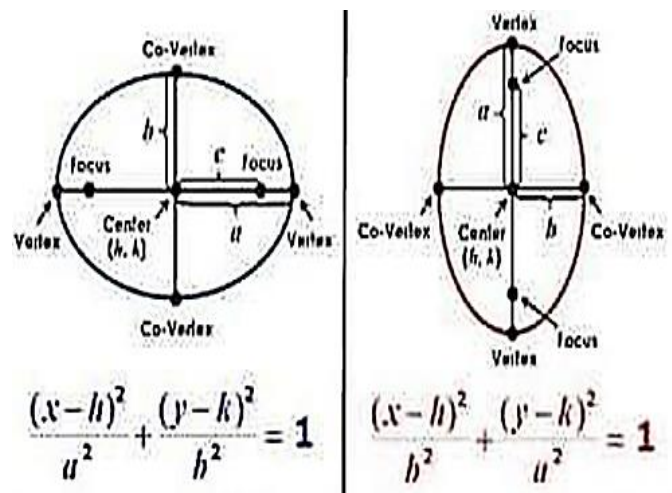
Fig 3 Diagram of Standard Equations for Ellipses

2.4 Hyperbola

A hyperbola is the set of points in a plane whose distances from two fixed points, called foci, in the plane have a constant difference [Larson (2013)].

The asymptotes of a hyperbola are two imaginary lines that the hyperbola is bound by.

Standard Equations for hyperbolas centered at (h,k) with axes parallel to the coordinate axes are as follows,

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \text{ (Line of foci parallel to } x\text{-axis)} \qquad (2)$$

- Vertices: $(h \pm a, k)$
- Foci: $(h \pm \sqrt{a^2 + b^2}, k)$
- Asymptotes: $(y-k) = \pm(b/a)(x-h)$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 \text{ (Line of foci parallel to } y\text{-axis)} \qquad (3)$$

- Vertices: $(h, k \pm a)$
- Foci: $(h, k \pm \sqrt{a^2 + b^2})$
- Asymptotes: $(y-k) = \pm(a/b)(x-h)$

Fig 5 Diagram of Standard Equations for Hyperbolas

3. APPLICATIONS OF CONIC SECTIONS

Conic sections are useful in every place of modern society. Conic sections can be found in most things that we encounter daily. Ro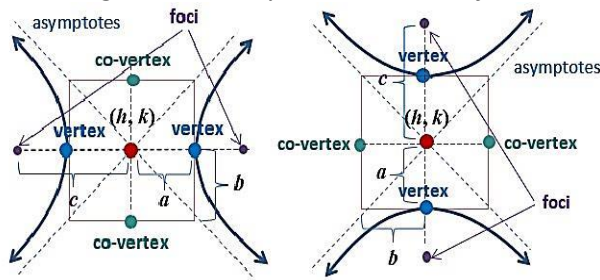ller coasters, arches, suspension bridges are the shapes of parabola. Footballs, tunnels, our eyes are the shapes of ellipse. Pizza, Ferris wheels, clocks are circular shapes. Nuclear cooling towers are hyperbolic shapes. The use of conic sections in civil engineering, electronics and communication engineering and mechanical engineering are described in the following applications.

3.1 Application in Civil Engineering

Civil engineers always calculate the height and width of buildings, arches, towers, pagodas and so on before they build. The following problems show calculating the vertical lengths of Nagayon Mahabodhi pagoda's arch from our native town, Meiktila by using the parabolic equation and computing the width of natural draft cooling tower at the top by using the hyperbolic equation.

3.1.1 Calculating the Vertical Lengths of Nagayon Mahabodhi Pagoda's Arch

Fig 6 Nagayon Mahabodhi Pagoda in Meiktila

When we went and worshiped to Nagayon Mahabodhi pagoda in our native town, Meiktila, we knew that the opening arch of the pagoda is the shape of the parabola. So, we determined to measure the width and height of the arch to calculate its vertical lengths by using the parabolic equation.
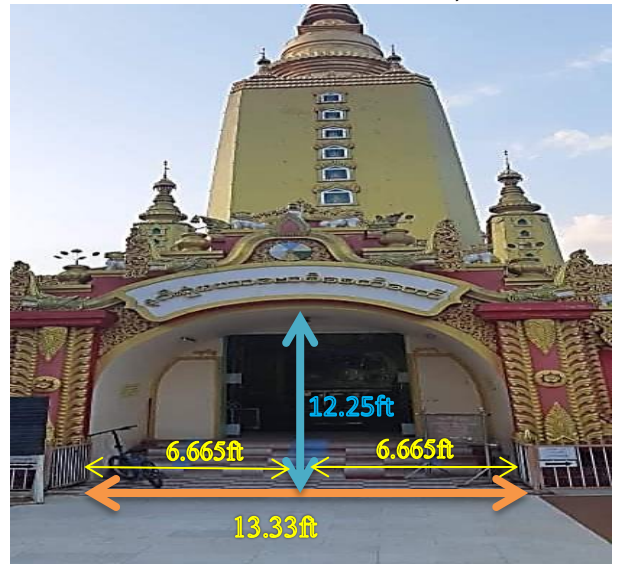
Fig 7 Parabolic Opening Arch

According to the giant tape measure, its height is 12.25 feet and its total width is 13.33 feet. Vertical lengths are the important parts of designing arch for civil engineers. The following calculations describe the use of parabolic equation in finding the vertical lengths of the arch.

The standard equation for parabola that opens downward is

$$(x-h)^2 = -4a(y-k)$$

In this case, vertex (h,k) is $(0,0)$. By substituting the half of the width, $x = 6.665$ ft and downward height, $y = -12.25$ ft in the above equation,

$$(6.665)^2 = -4a(-12.25)$$

$$44.422225 = 49a$$

$$a = \frac{44.42225}{49}$$

$$a = 0.907 \,.$$

Since $a = 0.907$, the parabolic equation becomes $x^2 = -3.628y$.

Fig 8 Parabolic Equation of Nagayon Pagoda's Arch

Now, the vertical lengths of the arch can be specified. The vertical length of the arch at a horizontal distance 2ft from the center can be calculated as follows,

$$(x-h)^2 = -4a(y-k)$$

$$(2)^2 = -4(0.907)y$$

$$y = -\frac{4}{3.628}$$

$$y = -1.103 \text{ ft.}$$

To get the vertical length at a horizontal distance 2ft from the center, the result y is subtracted from the total height. Therefore, the vertical length at a horizontal distance 2ft from the center $= -12.25\text{ft} - (-1.103\text{ft}) = -11.147\text{ft}$.

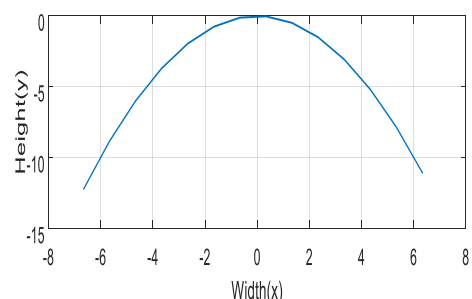The vertical length of the arch at a horizontal distance 4ft from the center can be calculated as follows,

$$(x-h)^2 = -4a(y-k)$$

$$(4)^2 = -4(0.907)$$

$$y = -\frac{16}{3.628}$$

$$y = -4.41 \text{ ft.}$$

To get the vertical length at a horizontal distance 4ft from the center, the result y is subtracted from the total height. Therefore, the vertical length at a horizontal distance 4ft from the center $= -12.25\text{ft} - (-4.41\text{ft}) = -7.84\text{ft}$.
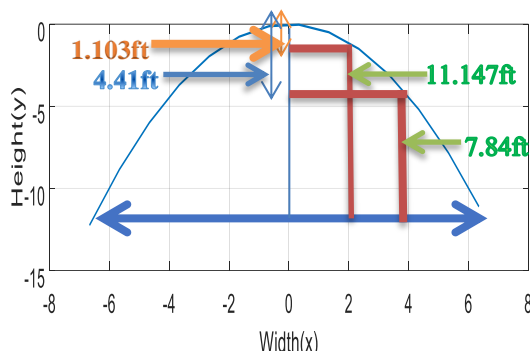
Fig 9 Vertical Lengths of the Arch of Nagayon Pagoda

Likewise, the other vertical lengths of the arch can be calculated by substituting the horizontal distances to the standard equation for parabola that opens downward.

Eventually, we learned that civil engineers built arches by using the parabolic equations.

3.1.2 Calculating the Width of Natural Draft Cooling Tower at the Top
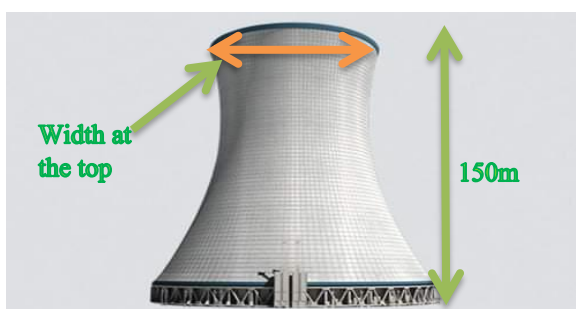
Fig 10 Hyperbolic Natural Draft Cooling Tower

A civil engineer designs a hyperbolic natural draft cooling tower that is 150m high for more efficient cooling of power plants.
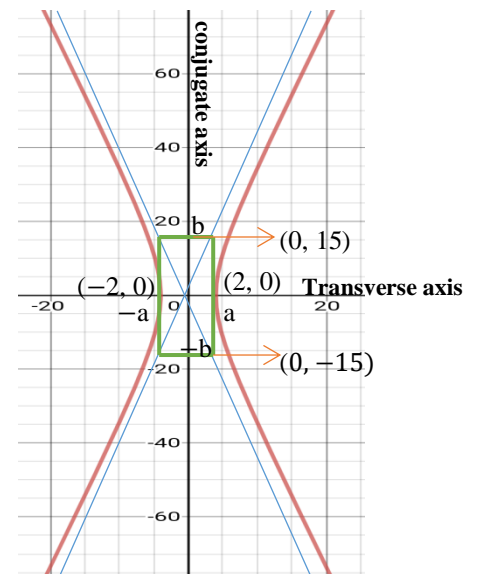
Fig 11 Sketch Diagram of Hyperbolic Cooling Tower with Data

In this case, the natural draft cooling tower is the shape of the hyperbola and therefore, the corresponding standard hyperbolic equation is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

where the center (h,k) is $(0,0)$. x means the half of the width and desired width at the top can be represented as $2x$. y represents the half of the height of the tower and therefore, $y = 75$. a represents the distance from the center $(0,0)$ to the vertex. From the figure 11, the vertices are $(2,0)$ and $(-2,0)$. Thus, $a = 2$. b represents the distance from the center to the endpoint of the conjugate axis and therefore, $b = 15$. The transverse axis is a line segment that passes through the center of the hyperbola and has vertices as its endpoints. The conjugate axis is perpendicular to the transverse axis and has the co-vertices as its endpoints.

According to the above data, the hyperbolic tower can be modeled by the equation, $\frac{x^2}{16} - \frac{y^2}{225} = 1$, where the units are in meters. The width of the tower at the top can be specified as follows.

$$\frac{x^2}{16} - \frac{75^2}{225} = 1$$

$$\frac{x^2}{16} - 25 = 1$$

$$\frac{x^2}{16} = 26$$

$$x = \sqrt{416}$$

$$x = 20.396 \text{ m.}$$

But, the final result x is the half of the width. So, the width of the tower at the top $= 2x = 2(20.396) = 40.792m$.

Fig 12 Hyperbolic Equation of Natural Draft Cooling Tower

A cooling tower is a heat rejection device that rejects waste heat to the atmosphere through the cooling of a water stream to a lower temperature. Because of their hyperbolic form, these structures are able to withstand extreme winds. In designing and constructing, the standard form of hyperbolic equations are very important parts of role because the measurements must be correct and if something goes wrong, the tower will not b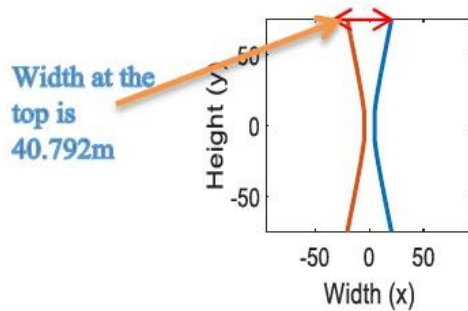e able to withstand. In this problem, the width at the top can be accurately computed by calculating the other measurements with the standard form of hyperbolic equation.

3.2 Application in Electronics and Communication Engineering

Radio transmitter is a part of the electronics and communication engineering. The following problem describes calculating the distance to pick up signal from a small transmitter by using the standard form of circle equation.

An electronics and communication engineer designs a small radio transmitter that broadcasts signals through a circular region in a 12 miles radius. A person drives along a straight line from a city, 7 miles north and 17 miles west of transmitter to a second city, 7 miles north and 14 miles east of the transmitter. An electronics and communication engineer can calculate the distance to pick up a signal from the transmitter for a driving person as follows.
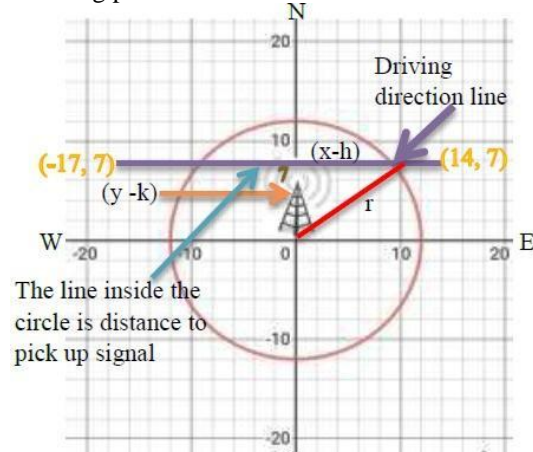
Fig 13 Transmission Region of the Small Radio Transmitter

The transmission region of the small radio transmitter is 12 miles radius. So, $r = 12$. The center (h, k) is $(0,0)$. The person drives to north 7 miles twice. So, $y = 7$. The distance to pick up a signal from the transmitter can be calculated as follows,

The standard form of circle equation is
$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-0)^2 + (7-0)^2 = (12)^2$$
$$x^2 + 49 = 144$$
$$x^2 = 144 - 49 = 95$$
$$x = \sqrt{95} = 9.75 \text{ miles.}$$

The distance to pick up a signal inside the circular transmission region can be specified as $2x = 2(9.75) = 19.5$ miles.
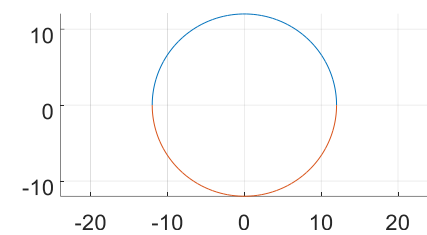
Fig 14 Transmission Region of the Small Radio Transmitter

3.3 Application in Mechanical Engineering

A mechanical engineer designs a truck that is 12 feet wide and 10 feet high. The entrance to a tunnel over a one-way road is half an ellipse with a height of 12 feet and a width of 36 feet. He can calculate whether his designing truck could or could not pass through the tunnel by using the standard form of ellipse equation.
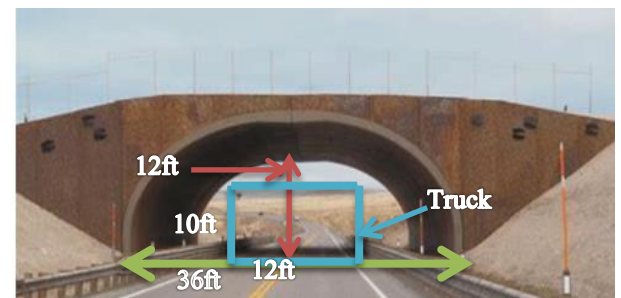
Fig 15 Elliptical Tunnel with Truck

The standard form of ellipse equation for major axis (horizontal) is as follows,

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$
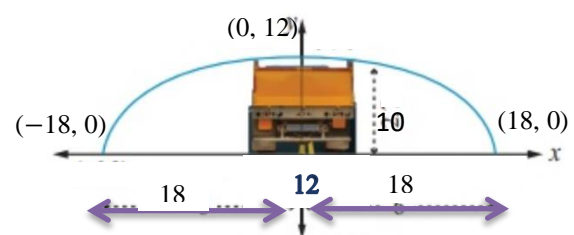
Fig 16 The Sketch of Elliptical Tunnel with Truck

From the given data, the center (h,k) is $(0,0)$. Since the width of the truck is 12ft, x is 6ft. The height of the truck, y is 10ft. Since the width of the tunnel is 36ft, a is 18ft. The height of the tunnel, b is 12ft.

Firstly, the left side of the equation is solved to know that a truck clears or does not clear the tunnel.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2}$$

$$= \frac{(6-0)^2}{18^2} + \frac{(10-0)^2}{12^2}$$

$$= \frac{36}{324} + \frac{100}{144} = \frac{29}{36} = 0.81 < 1.$$

According to the calculation of standard form of ellipse equation, the maximum size of the truck should be no more than 36ft wide and 12ft high so that it can easily pass through the tunnel. If the size of the truck is more than the size of the tunnel, the result of the left side of the ellipse equation will be greater than 1. Therefore, the truck cannot pass through the tunnel.

4. CONCLUSIONS

Conic sections are in many objects that are used in everyday life. Many things are shaped in parabolas, circles, ellipses and hyperbolas. Parabolic satellite antennas, suspension bridges and arches look like the parabola. Motor cycles' wheels, fans and clocks are the circular shapes. Elliptical tunnels, footballs and planets' orbits are the shapes of ellipse. Natural draft cooling towers and hyperbolic lamps are the shapes of hyperbola. So, conic sections are seen in everywhere.

In this paper, the applications of conic sections are widely presented in engineering fields such as civil engineering, electronics and communication engineering and mechanical engineering. The standard equations of conic sections are very effective in solving the engineering problems. These equations are also useful to solve the problems of other fields such as medicine, science and sociology, etc. Therefore, the conic sections are very important in real life.

ACKNOWLEDGEMENT

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Some Applications of Hamiltonian Graphs

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ABSTRACT: This paper provides two objectives. The first objective is introduced by some basic concepts and terms in graph theory. The second objective is illustrated with some applications by using graph theory to solve real life problems. This paper presents the problem of determining whether a given graph contains a cycle through each of its vertices, known as Hamiltonian cycle. Hamiltonian path is a subgraph of a Hamiltonian cycle and every graph that has a Hamiltonian cycle also has a Hamiltonian path. And then, the directed graphs, which edges have directions are considered in this paper. Directed graphs are required in many practical situations. The directed graph play an important role in the study of sequential machines. Finally, three applications of graph theory: the traveling salesman problem, gray codes and a job sequencing problem, related to Hamiltonian cycle and directed Hamiltonian path are presented.

KEYWORDS: *simple graph, directed graph ,cycle, path, Hamiltonian graph*

1. INTRODUCTION

All graphs in this paper are simple finite graphs. We use standard terminology notation from [1] except as indicated. For any graph G, we use V (G) and E (G) to denote the set of vertices and the set of edges of G, respectively. The degree of vertex v is denoted by d(v). In this paper we consider the problem of determining whether a given graph contains a cycle through each of its vertices, that is Hamiltonian cycle. Hamiltonian path and cycle are named after Irish mathematician Sir William Rowan Hamilton (1857) who invented the icosian game, also known as Hamilton's puzzle. The development of the theory of Hamiltonian graph has seen a series of results based on controlling the degree of vertices of G. Theorem(Ore) stated that if for each pair of nonadjacent vertices u and v of a graph G with n vertices, $n \geq 3$, $d(u)+d(v) \geq n$, then G has a Hamiltonian cycle. Theorem (Dirac) stated that if G is simple graph with n vertices, $n \geq 3$ such that the degree of every vertex in G is at least n/2 then G has a Hamiltonian cycle. Both Theorem (Ore) and Theorem (Dirac) provide sufficient conditions for a graph to be Hamiltonian cycle.

2. SOME BASIC CONCEPTS ON GRAPHS

2.1 Definitions

A **graph** G = (V, E) consists of a finite set V of vertices and a set E of edges such that each edge connects two vertices. A **loop** is an edge whose two end points are the same. A **simple graph** is a graph with no loops and parallel edges. The **complete** graph on n vertices for n≥1, which we denote K_n, is a graph with n vertices and an edge joining every pair of distinct vertices. A **directed graph** D = (V, A) consists of a non empty set V of vertices and a set A of ordered pairs of vertices. A **walk** of a graph G is an alternating sequence of vertices and edges $v_0, e_1, v_1, \ldots, v_{n-1}, e_n, v_n$, where v_0 and v_n are called the origin and terminus of walk. If all vertices of a walk are distinct, then it is called a **path**. A **cycle** in a graph G is a path that begins and ends at the same vertex, i.e., a closed path of non-zero length that does not contain a repeated edge. A graph with number on an edges is called a **weighted graph**. If edge e is labeled k, then the weight of edge e is k. The **n-cube graph**, denoted Q_n, whose vertex set consists of all bit strings of length n and whose edge set consists of all unordered pairs of strings that differ in exactly one position [1], [3].

2.2 Theorem (Dirac)

If G is a simple graph with n vertices, $n \geq 3$ such that the degree of every vertex in G is at least n/2, then G has a Hamiltonian cycle.

Proof: Suppose that G is not Hamiltonian. Let G be a maximal non-Hamiltonian simple graph with $n \geq 3$ and $d(v) \geq n/2$. Since $n \geq 3$, G cannot be complete. Let u and v be nonadjacent vertices in G. By the choice of G, G + uv is Hamiltonian. Moreover, since G is non-Hamiltonian, each Hamiltonian cycle of G + uv must contain the edge uv. Thus, there is a Hamiltonian path $v_1, v_2, \ldots, v_n$ in G with origin $u = v_1$ and terminus $v = v_n$. Set $\{v_i / uv_{i+1} \in E\}$ and $T = \{v_i / v_i v \in E\}$ where $i = 2, 3, \ldots, n-1$. So, the number of vertices in S is d(u)

and the number of vertices in T is d(v). Therefore, $\left|S\right| = d(u)$ and $\left|T\right| = d(v)$.

Then $v_n \notin S$, $v_n \notin T$.

Since $v_n \notin S \cup T$, we have

$$\left|S \cup T\right| < n \qquad (1)$$

Furthermore $|S \cap T| = 0$ (2)

Since if $|S \cap T|$ contained some vertex v_i, then G would have the Hamiltonian cycle $v_1, v_2 \ldots v_i v_n v_{n-1} \ldots v_{i+1} v_1$, contrary to assumption .

Using (1) and (2) we obtain

$$d(u) + d(v) = |S| + |T| = |S \cup T| + |S \cap T| < n \qquad (3)$$

This is a contradiction, because $d(v) \geq n/2$ for all v in G, and so $d(u) + d(v) \geq n/2 + n/2$ giving $d(u) + d(v) \geq n$. Hence, G has a Hamiltonian cycle [4].

2.3 Example

Figure 1 demonstrates the result of Theorem 2.2. In this graph, $d(v_i) \geq n/2$, for each i =1, 2, …, 6, so it has a Hamiltonian cycle $v_1, v_3, v_2, v_4, v_5, v_6, v_1$.

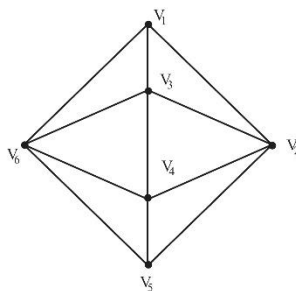Fig 1 Simple graph G

2.4 Theorem (Ore)

If G is a simple graph with n vertices, n $\geq$ 3 such that $d(u) + d(v) \geq n$ for every pair of nonadjacent vertices u and v in G, then G has a Hamiltonian cycle.

Proof: Suppose for contradiction. Let G satisfies the condition and G is not Hamiltonian. Adding one edge to G until the resulting graph G' = (V, E') would be Hamiltonian. Take any x, y $\in$ V, xy $\notin$ E'.

$x = v_1, v_2, \ldots, v_{n-1}, v_n = y$ is a Hamiltonian path in G'. For all vertices v_i is adjacent to x then v_{i-1} cannot be adjacent to y, $2 \leq i \leq n$. Otherwise $x = v_i, v_{i+1} \ldots, y$, $v_{i-1}, v_{i-2}, \ldots, v_2, x$ is a Hamiltonian cycle in G'.

$$d(y) \leq n-1 - d(x)$$

$d(n) + d(y) \leq n - 1$ which contradicts the hypothesis. Hence G has a Hamiltonian cycle [4].

2.5 Example

In the graph G of figure 2, we can see that the sum of the degree of distinct nonadjacent vertices is always n. Thus, G has a Hamiltonian cycle $v_1, v_4, v_3, v_2, v_5, v_1$.
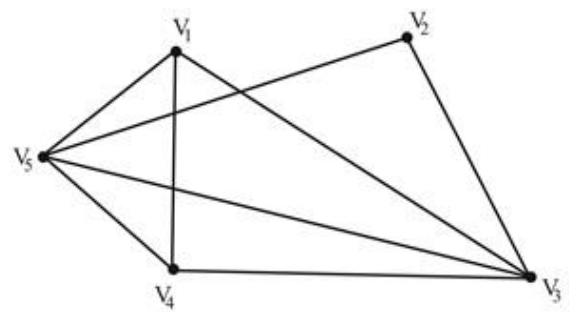
Fig 2 Simple graph G

3. APPLICATIONS OF HAMILTONIAN PATH AND CYCLES

3.1 Hamiltonian graphs

A path that contains every vertex of G is called a **Hamiltonian path** of G. A **Hamiltonian cycle** of a graph G is a cycle that contains every vertex of G. A graph is **Hamiltonian** if it contains a Hamiltonian cycle.

A **directed Hamiltonian path** of D is directed path that includes every vertex of D. A **directed Hamiltonian cycle** of D is a directed cycle that includes every vertex of D.

3.2 A Job Sequencing Problem

A number of jobs $J_1, J_2 \ldots, J_n$ have to be processed on one machine; for example, each J_i might be an order of bottles or jars in a glass factory. After each job, the machine must be adjust to fit the requirements of the next job. If the time of adaptation from job J_i to job J_j as t_{ij} , find a sequencing of the jobs that minimizes the total machine adjustment time.

We can solve this problem by constructing a directed graph D and finding a directed Hamiltonian path.

Step 1 Construct a directed graph D with vertices $v_1, v_2 \ldots, v_n$, such that $(v_i, v_j) \in A$ if and only if $t_{ij} \leq t_{ji}$.

Step 2 Find a directed Hamiltonian path $(v_{i1}, v_{i2}, \ldots, v_{in})$ of D and sequence the jobs accordingly.

As an example, suppose that there are six jobs J_1, J_2, J_3, J_4, J_5 and J_6 and that the adjustment matrix is

	J_1	J_2	J_3	J_4	J_5	J_6
J_1	0	6	4	5	3	2
J_2	2	0	2	3	4	3
J_3	3	6	0	2	3	4
J_4	2	5	5	0	2	3
J_5	2	4	5	6	0	6
J_6	5	5	3	4	2	0

Fig   3
Directed graph D

A B E D C A	196
A B E C D A	188
A B D E C A	208
A B D C E A	161
A D B C E A	195
A D B E C A	228
A D E B C A	230
A D C B E A	183
A C D B E A	189
A C B D E A	203
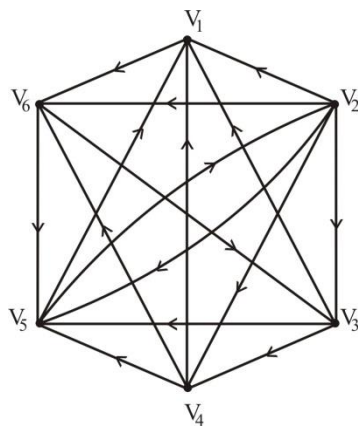
The sequence $J_1 \to J_2 \to J_3 \to J_4 \to J_5 \to J_6$ requires 18 units in adjustment time. To find a better sequence, construct the directed graph D as in step 1.

$(V_1, V_6, V_5, V_2, V_3, V_4)$ is a directed Hamiltonian path of D, and yields the sequence $J_1 \to J_6 \to J_5 \to J_2 \to J_3 \to J_4$ which requires only 12 units of adjustment time. We note that the reverse sequence

$J_4 \to J_3 \to J_2 \to J_5 \to J_6 \to J_1$ is not optimal, which requires 26 units of adjustment time [1].

3.3 The Travelling Salesman Problem

A travelling salesman wants to visit a number of towns and then return to his starting point. He can travel to every town exactly once and return to the starting point in any order. According to the order, we can find shortest total distance which is known as the travelling salesman problem. In graphical terms, the aim is to find a minimum weight Hamiltonian cycle in a weighted complete graph. We shall call such a cycle an optimal cycle.

Suppose that the salesman wants to visit the five cities A, B, C, D and E (see Fig 4). Representing the cities by vertices and the roads between them by edges. We can draw a graph. To solve this problem we can assume the salesman starts in A and examine all possible ways for him to visit the other four cities and then return to A. There are a total of 24 such cycles, but since we travel the same distance when we travel a cycle in reverse order. We need only consider 12 different cycles to find the minimum total distance he must travel. As can be seen from the table 1, the minimum total distance of 161 miles is traveled using the cycle A E C D B A (or its reverse).

Table 1. Given route and total distance for travelling salesman problem

Route	Total distance (miles)
A B C D E A	163
A B C E D A	202
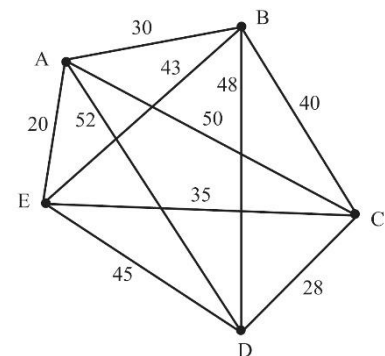
Fig 4 Graph model for travelling salesman problem

Now we shall examine the number of cycles to solve the problem if there are n vertices in the graph. Once a starting point is chosen, there are (n−1)! different Hamiltonian cycles to examine, since there are n−1 choices for the second vertex, n−2 choices for the third vertex and so on. Since a Hamiltonian cycle can be traveled in reverse order. Theoretically, the problem of the travelling salesman can always be solved by enumerating all $\frac{(n-1)!}{2}$ Hamiltonian cycle, calculating the distance traveled in each, and then choosing the minimum total distance [2].

3.4 Example
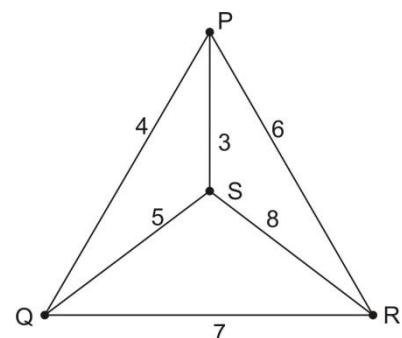
Fig 5 Weighted complete graph G

Figure (5) represent the weighted complete graph having four vertices and six edges. We can solve the travelling salesman problem for this graph by finding the total weight of all Hamiltonian cycles and determining a cycle with minimum total weight.

The total number of different Hamiltonian cycles

$$= \frac{(n-1)!}{2} = \frac{(4-1)!}{2} = 3$$

The Hamiltonian cycles that starts with P and give its weight are:

P S R Q P = 3+8+7+4 = 22

P R S Q P = 6+8+5+4 = 23

P R Q S P = 6+7+5+3 = 21

Hence, the total minimum weight is 21 units.

3.5 Gray Codes

The position of a rotating pointer can be represented in digital form. One way to do this is to split the circle into 2^n arcs of equal length and to assign a bit string of length n to each arc. Two ways to do this using bit strings of length three are shown in Fig 6.

The digital representation of the position of the pointer can be determined using a set of n contacts. Each contact is used to read one bit in the digital representation of the position.

When the pointer is near the boundary of two arcs, a mistake may be made in reading its position. This may result in a major error in the bit string read. For instance, in the coding scheme in Fig 6 (a), if a small error is made in determining the position of the pointer, the bit string 100 is read instead of 011. All three bits are incorrect. To minimized the effect of an error in determining the position of the pointer, the assignment of the bit strings to the 2^n arcs should be made so that only one bit is different in the bit strings represented by adjacent arcs. This is exactly the situation in the coding scheme in Fig 6 (b). An error determining the position of the pointer gives the bit string 010 instead of 011. Only one bit is incorrect.

(a)   (b)
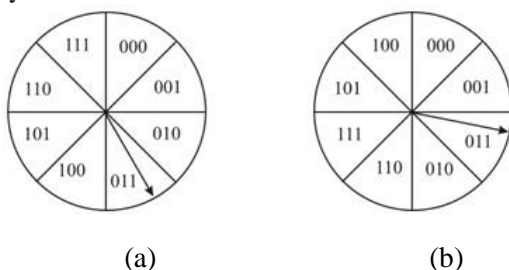
Fig 6 Converting the position of a pointer (a) into digital form (b)

A Gray code is a labeling of the arcs of the circle so that adjacent arcs are labeled with bit strings that differ in exactly one bit. The assignment in Fig 6 (b) is a Gray code. We can find a Gray code by listing all bit strings of length n in such a way that each string differs in exactly one position from the preceding bit string, and the last string differs from the first in exactly one position. We can model this problem using the n-cube

Q_n. Hamiltonian cycle in Q_n is required to solve this problem. Such Hamiltonian cycles are easily found.
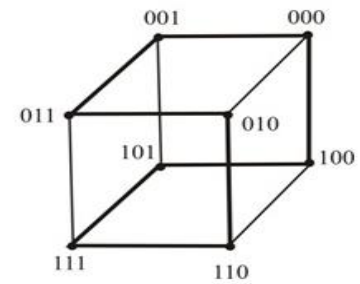
Fig 7 A Hamiltonian cycle of Q_3

For instance, a Hamiltonian cycle for Q_3 is illustrated in Fig 7. The sequence of bit strings differing in exactly one bit produced by this Hamiltonian cycle is 000, 001, 011, 010, 110, 111, 101, 100, 000 [4].

Gray codes are named after Frank Gray, who invented them in the 1940s at AT & T Bell Laboratories to minimize the effect of errors in transmitting digital signals.

4. CONCLUSIONS

Hamiltonian paths and cycles can be used to solve real life problems. Many applications require a path or cycle that visits each road intersection in a city, each place pipelines intersect in a utility grid, or each vertex in a communications network exactly once. We can find a Hamiltonian path or cycle in the appropriate graph model can solve such problems.

ACKNOWLEDGEMENT

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Application of Triangular Elements Method for Poisson's Equation

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ABSTRACT*:* In this paper, we introduced the finite element idealization and triangular element idealization. And then, especially triangular element method for Poisson's equations are thoroughly discussed with boundary conditions and compared with the finite element solutions, finite different solutions and exact solutions of Poisson's equations. It is the purpose to present brief derivations of the finite element approximation describing a discrete model and wrote the computer program of triangular elements solution by
 using the C++.

KEYWORDS: *Finite Element Method, Triangular Element Method, Finite Different Method, Variational Formulation, C++.*

1. INTRODUCTION

Finite element method is summation of a piecewise elements in total element E approximation to the exact solution. Let us consider the differential problem $L\phi = f$ in R, with $B\phi = g$ on C, the bounding C subset R. In this problem, each element $\emptyset^e$ *(x, y)* approximations to the exact solution ϕ*(x, y)* is $\phi(x, y) = \sum_e \phi^e(x, y)$ where the summation is total elements E,(e = 1, 2, ---, E) and outside of the element is zero. $\emptyset^e(x, y) = N^e(x, y)\delta^e$ where $N^e(x, y)$ is called the shape function and $\delta^e \{\emptyset_1, \emptyset_1, \emptyset_2, \emptyset_3, ...\}$ is called the nodal displacement. Variational formulation[1] of Poisson's equation with boundary condition is

$$I[\phi] = \sum_e \iint_{R^e} \left\{ k\left(\frac{\partial\emptyset}{\partial x}\right)^2 + k\left(\frac{\partial\emptyset}{\partial y}\right)^2 2\emptyset f \right\} dx\, dy +$$
$$\sum_e \int_{C_2^e} \{\sigma(\emptyset^e)^2 - 2\emptyset^e h\}\, ds.$$

2.TRIANGULARELEMENTS DESCRETIZATION

Rectangular elements are not appropriate on the irregular boundaries because it is not easy to by the rectangular shape to the boundary geometry with elements as show in Figure 1. But triangular element is closely to the boundary than rectangular elements by a polygonal arc as show in Figure 2. Therefore, triangular element approximation is better than rectangular element approximation. If the mesh points are larger number, the accuracy of solution is more and more better. [1]
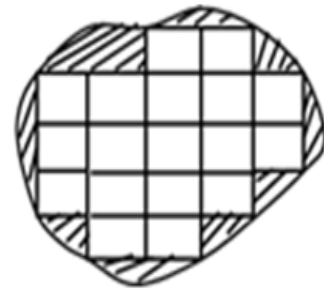
Fig1. A typical geometry suitable for approximation with rectangular elements
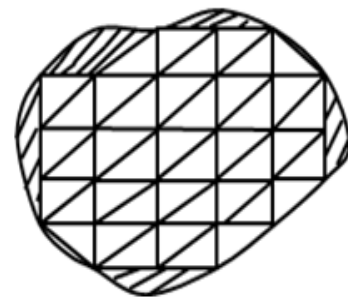
Fig 2. Approximation with triangular elements using the same number of nodes.

Consider the triangular element of the position P with the global coordinates *(x, y)* and local coordinates or triangle coordinates or area coordinates *(L_1, L_2, L_3)* as shown in Figure 3.
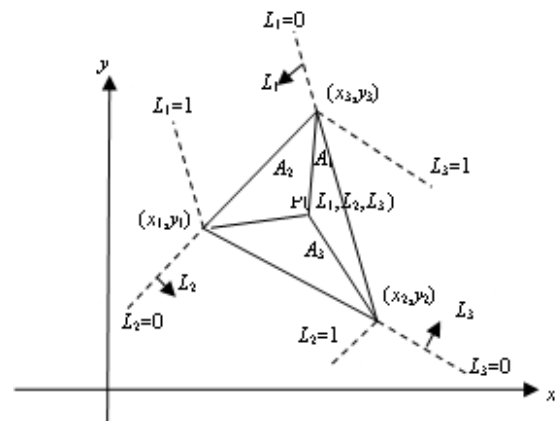
Fig 3. Area coordinates for a triangular element

$$L_1 = \frac{A_1}{A}, \quad L_2 = \frac{A_2}{A}, \qquad L_3 = \frac{A_3}{A}, \qquad (1)$$

where A_1, A_2, A_3 are the areas in the triangle A as shown in Figure 3 but three coordinates are not independent and they satisfied the equation

$L_1 + L_2 + L_3 = 1.$ (2)

The relation between the global coordinates (x, y) and the triangular coordinates (L_1, L_2, L_3) is

$x = L_1 x_1 + L_2 x_2 + L_3 x_3$ (3)
$y = L_1 y_1 + L_2 y_2 + L_3 y_3$ (4)

and it is obtained L_1 in terms of x and y is

$L_1 = \frac{a_1 + b_1 x + c_1 y}{2A}$ (5)

where the area A is given by

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$ (6)

and x_i, y_i are nodal coordinates and

$a_1 = x_2 y_3 - x_3 y_2$, $a_2 = x_3 y_1 - x_1 y_3$
$b_1 = y_2 - y_3$, $b_2 = y_3 - y_1$
$c_1 = x_3 - x_2$, $c_2 = x_3 - x_1$,...   (7)

The derivatives of L_i are

$\frac{\partial L_i}{\partial x} = \frac{b_i}{2A}$ and $\frac{\partial L_i}{\partial y} = \frac{c_i}{2A}$ (8)

where a_i, b_i and c_i are constants.

The integral involving the area coordinates [1] is.

$\iint_A L_1^m L_2^n L_3^p dx dy = \frac{2Am!n!p!}{(m+n+p+2)!}$ (9)

An element has three nodes with one degree of freedom at each node, the displacement variation throughout the element is linear form.

$\emptyset^e(x, y) = a_0 + a_1 x + a_2 y$ (10)

By using the nodal displacement and interpolating through the element gives

$\emptyset^e(x, y) = N^e(x, y)\delta^e$
$\emptyset^e(x, y) = [L_1 \quad L_2 \quad L_3]\{\emptyset_1 \quad \emptyset_2 \quad \emptyset_3\}$ (11)

The element stiffness matrix is given by

$k_{ij}^e = \iint_A k\left(\frac{\partial L_i}{\partial x} \frac{\partial L_j}{\partial x} + \frac{\partial L_i}{\partial y} \frac{\partial L_j}{\partial y}\right) dx dy$

$k_{ij}^e = \iint_A k\left(\frac{b_i b_j}{4A^2} + \frac{c_i c_j}{4A^2}\right) dx dy$ (12)

In the special case $k = 1$,

$$k_{ij}^e = \frac{1}{4A}(b_i b_j + c_i c_j)$$

The element force vector is given by

$f_i^e = \iint_A L_i f(x, y) dx dy$ (13)

terms of L_1, L_2, L_3 and hence f_i^e may be obtained. If the element ⎯ dary side 1-2 as show in Figure 4.
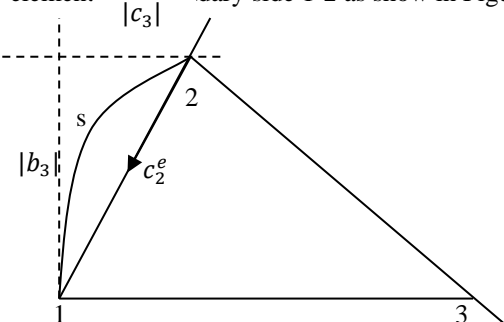
Fig 4. Boundary element with side 3-1 approximating the boundary

$L_3 = 0$ and $s = (b_3^2 + c_3^2)^{1/2} L_1$
so that $ds = (b_3^2 + c_3^2)^{1/2} dL_1$
since $L_2 = 1 - L_1$

$\bar{k}^e = \int_{c_2} \sigma(s) N^{e^t} N^e ds$ (14)

$\bar{f}^e = \int_{c_2} h(s) N^{e^t} ds$ (15)

$$\bar{k}^e = \int_0^1 \sigma \begin{bmatrix} L_1^2 & L_1 - L^2 & 0 \\ L_1 - L_1^2 & (1-L_1)^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} (b_3^2 + c_3^2)^{1/2} dL_1.$$ (16)

$\bar{f}^e = \int_0^1 h\{L_1 \quad 1-L_1 \quad 0\}(b_3^2 + c_3^2)^{1/2} dL_1$ (17)

These results will now be used to obtain a two-element solution using a single rectangular element.

3. TRIANGULAR ELEMENTS SOLUTION FOR POISSON's EQUATION

Consider the problem $-\nabla^2 \phi = 2(x + y) - 4$ in the square whose vertices are at $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$. The boundary condition are $\phi(0,y) = y^2$, $\phi(1,y) = 1-y$, $\phi(x,0) = x^2$, $\phi(x,1) = 1-x$.
The problem has symmetry about the line
$y = x$, $\frac{\partial \phi}{\partial n} = 0$
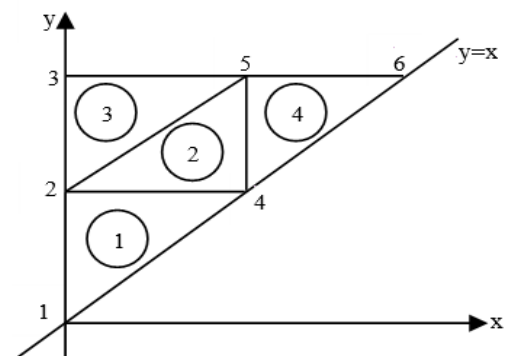
Fig 5. Four triangular element

The element stiffness matrices may be obtained directly from equation (12) and area of each triangle 1/8. Then

$$k^1 = \begin{matrix} & 1 & 4 & 2 & \\ \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/2 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix} & \begin{matrix} 1 \\ 4 \\ 2 \end{matrix} \end{matrix}$$

$$k^2 = \begin{matrix} & 5 & 2 & 4 & \\ \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/2 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix} & \begin{matrix} 5 \\ 2 \\ 4 \end{matrix} \end{matrix}$$

$$k^3 = \begin{matrix} & 2 & 5 & 1 & \\ \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/2 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix} & \begin{matrix} 2 \\ 5 \\ 1 \end{matrix} \end{matrix}$$

$$k^4 = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/2 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix}\begin{matrix}4\\6\\5\end{matrix}$$

The element force vector is given by equation (13) as

$$f_i = \iint_A L_i[2(x+y)-4]dxdy, \quad i=1,2,3$$

where 1, 2, 3 is the local nodal numbering defined in Fig 3. Then using equations (3) and (4)

$$f_i = \int_A L_i[2L_1(x_1+y_1)+2L_2(x_2+y_2)+2L_3(x_3+y_3)-4]\,dxdy$$

$$f_i = \frac{(x_i+y_i)}{24}+\frac{(x_j+y_j)}{48}+\frac{(x_k+y_k)}{48}-\frac{1}{6}$$

The only node without an essential boundary condition is node 4, thus only the equation associated with node 4 is assembled. This equation is

$$\sum_{j=1}^{6} K_{1j}\ \phi_j = F_4$$

where K and F are the over-all stiffness and force matrices respectively. Row 4 of K is [0 -1 0 2 -1 0]: and $F4 = f_2^1 + f_3^2 + f_1^4$, where the subscribe refer to the local nodal numbers of Figure3.

$$f_2^1 = (\tfrac{1}{2}+\tfrac{1}{2})/48+(0+\tfrac{1}{2})/48+(0+0)/48-\tfrac{1}{6}=-\tfrac{11}{96},$$

$$f_3^2 = (\tfrac{1}{2}+\tfrac{1}{2})/24+(\tfrac{1}{2}+1)/48+(0+\tfrac{1}{2})/48-\tfrac{1}{6}=-\tfrac{8}{96},$$

$$f_1^4 = (\tfrac{1}{2}+\tfrac{1}{2})/24+(1+1)/48+(\tfrac{1}{2}+1)/48-\tfrac{1}{6}=-\tfrac{5}{96}.$$

Therefore, $F = -\tfrac{1}{4}$

The essential boundary condition gives

$$\phi_1 = 0, \phi_2 = \frac{1}{4}, \phi_3 = 1, \phi_5 = \frac{1}{2}, \phi_6 = 0$$

so that

$$-\frac{1}{4}+2\phi_4-\frac{1}{2}=-\frac{1}{4}$$

which gives

$$\phi_4 = \frac{1}{4}$$

The solution at $(\tfrac{1}{4},\tfrac{1}{4})$, $(\tfrac{1}{4},\tfrac{3}{4})$ and $(\tfrac{3}{4},\tfrac{3}{4})$ is found by linear interpolation between nodes 1 and 4, 2 and 5 , 4 and 6 respectively.

4.FINITE DIFFERENT SOLUTION FOR POISSON's EQUATION

By using the finite different method for Poisson's equation
U(x+h,,y)+U(x-h,y)+U(x,y+h)+U(x,y-h)-4(x,y)
=h²f(x,y)

For P_{11}, $u_{21}+u_{01}+u_{12}+u_{10}-4u_{11} = 0.0625\{2(1/4+1/4)-4\}$

For P_{12}, $u_{22}+u_{02}+u_{13}+u_{10}-4u_{11} = 0.0625\{2(1/4+1/2)-4\}$

For P_{13}, $u_{23}+u_{03}+u_{14}+u_{12}-4u_{13} = 0.0625\{2(1/4+3/4)-4\}$

For P_{21}, $u_{31}+u_{11}+u_{22}+u_{20}-4u_{21} = 0.0625\{2(1/2+1/4)-4\}$

For P_{22}, $u_{32}+u_{12}+u_{23}+u_{21}-4u_{22} = 0.0625\{2(1/2+1/4)-4\}$

For P_{23}, $u_{33}+u_{13}+u_{24}+u_{22}-4u_{23} = 0.0625\{2(1/2+3/4)-4\}$

For P_{31}, $u_{41}+u_{21}+u_{32}+u_{30}-4u = 0.0625\{2(3/4+1/4)-4\}$

For P_{32}, $u_{42}+u_{22}+u_{33}+u_{31}-4u_{32} = 0.0625\{2(3/4+1/2)-4\}$

For P_{33}, $u_{43}+u_{23}+u_{34}+u_{32}-4u_{33} = 0.0625\{231/4+3/4)-4\}$.

By using the Gauss-Seidel Method,

u_{11}= 0.203125, u_{12}= 0.2890625, u_{13}= 0.484375

u_{22}= 0.28125, u_{23}= 0.335925, u_{33}= 0.265625.

5.COMPARISON OF THE SOLUTIONS FOR POISSON'S EQUATION

In the above mention results compared with the corresponding results using triangular elements solutions, finite different solutions and exact solutions at the same point. If we want to find the another solution of another nodes, element solution is easy to know the solution of the another node but finite different solution is second time plot the mesh point and calculate the next time.

Table 1. Comparison of the Solutions for Poisson's Equation

(x ,y)	(1/4, 1/4)	(2/4, 2/4)	(1/4, 3/4)	(3/4,3/4)
Four triangular elements solutions	0.125	0.25	0.375	0.125
Finite difference solutions	0.203125	0.28125	0.483475	0.265625
Exact Solutions	0.094	0.25	0.25	0.281

6. C++ PROGRAM OF TRIANGULAR ELEMENTS SOLUTION FOR POISSON'S EQUATION

The C++ program for solving Poisson's equation, using four triangular elements are described.

```
#include<iostream.h>
#include<iomanip.h>
#define ENODE 3
#define maxN 6
ostream &print1x3(float x[3]);
ostream &print3x1(float x[3]);
ostream &print3x3(float x[3][3]);
void MVM(float M[maxN][maxN], float V[maxN], int
K, int L, float Y[maxN]);
void MIV(float M[maxN][maxN],int n);
void OutText(float M[6][6],float V[6],int r);
```

```
void TriK(float K[3][3], float f[3], float x[3], float
y[3]){
        float a[3],b[3],c[3],A;
        int i,j,k;
        a[0]=x[1] * y[2] - x[2] * y[1];
        b[0]=y[1] * y[2];
        c[0]=x[2] - x[1];

        a[1]=x[2] * y[0] - x[0] * y[2];
        b[1]=y[2] * y[0];
        c[1]=x[0] - x[2];

        a[1]=x[2] * y[0] - x[0] * y[2];
        b[1]=y[2] * y[0];
        c[1]=x[0] - x[2];

        a[2]=x[0] * y[1] - x[1] * y[0];
        b[2]=y[0] * y[1];
        c[2]=x[1] - x[0];

        A=0;
        for(i=0; i<3; i++){
                A += a[i]/2;
                j = (i+1) % 3;
                k=(i+2)%3;
                f[i] = (x[i]+y[i])/3.0 + (x[j]+y[j])/6.0
+ (x[k]+y[k])/6.0 - 4/3.0;
        }
        for(i=0;i<3;i++){
                f[i] *= A;
                for(j=0; j<3; j++)

        K[i][j]=1/4.0/A*(b[i]*b[j]+c[i]*c[j]);
        }
}
    int main(void){
        float x[maxN]={0,0,0,0.5,0.5,1.0}, xx[3],
        y[maxN]={0,0.5,1,0.5,1,1}, yy[3],f[3],

u[maxN][2]={{0,0},{0,0.25},{0,1.0},{1,0},{0,0.5},{0,
0}},
        K[ENODE][ENODE],

Ks[maxN][maxN],Fs[maxN],Un[maxN];

                int
elem[4][ENODE]={{0,3,1},{4,1,3}, {1,4,2},{3,5,4}};
        int n=maxN,en=4;
        int i,j,k;
        //Clear Assemblage Matrices
        for(i=0;i<n;i++)
        for(j=0;j<n;j++)
                Ks[i][j]=Fs[j]=0;

        for(i=0;i<en;i++){
                for(j=0;j<ENODE;j++){
                        xx[j]=x[elem[i][j]];
                        yy[j]=y[elem[i][j]];
                }
                //Element Stiffness Matrix
                TriK(K,f,xx,yy);
                print1x3(f) << endl;
                print3x3(K) << endl;
```

```
                //Assemble Stiffness and
Force Matrix
                for(j=0;j<ENODE;j++){

        for(k=0;k<ENODE;k++)

        Ks[elem[i][j]][elem[i][k]] += K[j][k];

        Fs[elem[i][j]] += f[j];

                }
        }
        cout<< "Assembled equations (Before
Boundary Condition)" << endl;
        OutText(Ks,Fs,n);
        //Modify Stiffness Matrix and Force
Vector
        for(i=0;i<n;i++){
                if(u[i][0]==0){
                        for(j=0;j<n;j++)
                                Ks[i][j]=0;
                                Ks[i][i]=1;

        Fs[i]=u[i][1];

                }
        }
        cout<< "Assembled
equations " << endl;
        OutText(Ks,Fs,n);
        //Inverse Matrix
        MIV(Ks,n);
        //Caluclate Displacement
Vector
        MVM(Ks,Fs,n,n,Un);
        cout<< "(.25,.25  .5,.5
.25,.75        .75,.75)" << endl;
        cout<< setw(9) <<
(Un[0]+Un[3])/2.0
                        << setw(9) <<
(Un[3]+Un[3])/2.0
                        << setw(9) <<
(Un[1]+Un[4])/2.0
                        << setw(9) <<
(Un[3]+Un[5])/2.0 << endl;
                return 0;
        }
    ostream &print1x3(float x[3]){
        int i;
        cout<< setprecision(3);
        for(i=0;i<3;i++)
                cout<< setw(8) << x[i] << " ";
        return cout;
    }
    ostream &print3x1(float x[3]){
        int i;
        cout<< setprecision(3);
        for(i=0;i<3;i++)
                cout<< setw(8) << x[i] << endl;
        return cout;
    }
    ostream &print3x3(float x[3][3]){
```

```
        int i;
        for(i=0;i<3;i++)
                print1x3(x[i])<< endl;
        return cout;
    }
    void OutText(float M[6][6], float V[6], int r){
        int i,j;
        cout<<setiosflags(ios::fixed);
        cout<<setprecision(4);
        cout<<endl;
        for(i=0;i<r;i++){
                for(j=0;j<r;j++)
                        cout<<setw(8)<< M[i][j]<<"
";
                cout<<" | " <<setw(8)<< V[i] << "
<== f" << i <<endl;
        }
    }
    void MVM(float M[maxN][maxN], float V[maxN],
int K, int L, float Y[maxN]){
        float X;
        int i,j;
        for(i=0; i<K ; i++){
                X=0.0;
                for(j=0; j<L ; j++)
                        X += M[i][j] * V[j];
                        Y[i]=X;
        }
    }
    void MIV(float M[maxN][maxN],int n){
        #define FORJN for(j=0;j<n;j++)
        #define FORKN for(k=0;k<n;k++)
        #define FORIN for(i=0;i<n;i++)
        #define M(i,j) M[(i)][(j)]

        int i,j,k;
        float KII;
        FORKN{
                KII=M(k,k);
                M(k,k)=1;
                FORJN
                        M(k,j)/=KII;
                FORIN{
                        if(i!=k){
                                KII=M(i,k);
                                M(i,k)=0;
                                FORJN
                                        M(i,j) -= -
M(k,j) * KII;
                        }}}}
```

Output of the element stiffness matrices

Output of the assembled equations (Before boundary conditions)

Above the mention result of The C++ program for solving Poisson's equation and finite element solution are the same. We using the C++ program, we are reduced to time limiting and easy to find the another solution.

7. CONCLUSIONS

In the above mentioned result, triangular elements solution is better than the finite different solution for Poisson's equation. And then, triangular elements solution and C++ program solution are the same for this problems. Finite element approximation for field problems improvements made by refining the finite element mesh. Higher-order elements are get a better polynomial approximation but integral involved would be necessary to use numerical integration. Using this program, we can easy to know the other points of this problem.

If we want to another solution or another point, finite element method solution is easy to replace at these points but finite different method is next time will mesh and calculate.

It is reasonable to expect that the proposed finite elements model is a very good approximation of the continuum. This method is now very widely used and forms the basis of most calculations

ACKNOWLEDGEMENT

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Some Applications of Projectile Motions in Real Life

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ABSTRACT: The purpose of this paper is to present the applications of fractional calculus to the projectile problems, hence, the features of the projectile motion such as maximum height, flight time and range will be described from the fractional calculus point of view. The basic concepts of vector-valued functions are discussed. The domains of these functions are real numbers but their ranges are vectors. In particular, let's consider the effect of gravity on the motion of an object as it travels through the air, and how it determines the resulting trajectory of that object. In this paper, we ignore the effect of air resistance. And then the paths and motions of objects moving in a plane or in space are described. The velocities and accelerations of these objects along their paths are calculated. The integrals of vector functions and their applications to motion along a path in space or in the plane are investigated.

KEYWORDS: *acceleration, maximum height, projectile motion, range, speed.*

1. INTRODUCTION

Vectors are used to study the analytic geometry of space, where they give simple ways to investigate lines, planes and curves in space. Using these geometric ideas to stuldy motion in space and the calculus of functions of several variables with their many important applications in engineering science [Jordan (2007)]. Derivatives and motion of the vectors are also discussed. Projectile motion expresses how an object fired at some angle from an initial condition, and based on by only the force of gravity, travels in a vertical coordinate plane [David (1988)]. We neglect the effects of any frictional along the object, which may change with its speed and altitude, and also the facts that the force of gravity changes slightly with the projectile's changing height. Besides, we neglect the long-distance effects of the earth turning under the projectile, such as in a rocket launch or the firing of a projectile from an archer [Naylor (1980)].

2. VECTOR-VALUED FUNCTIONS IN SPACE

When a particle moves through space during a time interval *I*, the particle's coordinates as functions defined on *I*:

$$x = f(t) \,,\, y = g(t) \,,\, z = h(t) \,,\, t \in I. \tag{1}$$

The points $(x, y, z) = (f(t), g(t), h(t))\,,\, t \in I$.

A **curve** in the space can also be represented in the vector form;

$$r(t) = f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k}, \tag{2}$$

where $r(t) =$ position vector.

The function *f, g,* and *h* are the **component functions** of the position vector.

The particle's path as the curve traced by *r* during the time interval *I*.

The position vector *r* as a vector function of the real variable t on the interval *I*. More generally, a vector-valued function or vector function on a domain set D is a rule that assigns a vector in space to each element in D. In Figure 1 displays several space curves generated by a computer graphing program [Thomas (1914)].

The graph of position vector is illustrated by using Matlab code.

```
t=0:0.01:6*pi;
subplot (1,1,1);
plot3(cos(t), sin(t), t,'--g*')
xlabel('x')
ylabel('y')
zlabel('z')
```
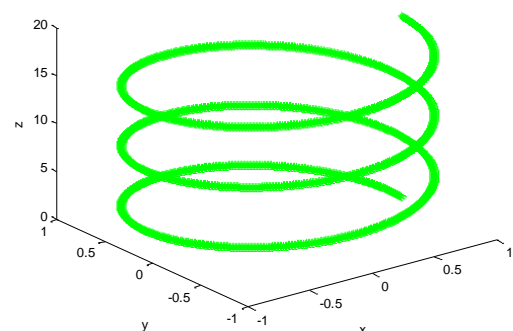

Fig 1 The Position Vector of $\dfrac{dr}{dt}$.

3. DERIVATIVES AND MOTIONS OF THE VECTORS

Suppose that $r(t) = f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k}$ is the position vector of a particle moving along a curve in space and that *f, g,* and *h* are differentiable functions of *t.* A vector function *r* is **differentiable** if it is differentiable at every point of its domain. The curve traced by *r* is the **smooth** if $\dfrac{dr}{dt}$ is continuous and never 0, that is, if *f , g* , and *h* have continuous first derivatives that are not simultaneously 0.

3.1. Definitions

If a particle moves along a smooth curve in space given by position vector $r(t)$, where *t* is time, then $v(t) = \dfrac{d\,r}{d\,t}$ is the particle's **velocity vector**, tangent to the curve. At any time *t*, the direction of *v* is the **direction of motion**, the magnitude of *v* is the particle's **speed**, and the derivative $a = \dfrac{d\,v}{d\,t}$ when it exists, is the particle's **acceleration vector** .

Velocity: $v(t) = \dfrac{d\,r}{d\,t}$.   (3)

Speed: $|v| = \dfrac{d\,\mathbf{s}}{d\,t}$.   (4)

Acceleration: $a = \dfrac{d\,v}{d\,t} = \dfrac{d^2 r}{dt^2}$.   (5)

Unit vector: $\dfrac{v}{|v|}$.   (6)

3.2. Example

A particle moves in space is defined by the position vector $r(t) = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + 2\cos^2 t\,k$. The velocity, acceleration, and speed of a particle whose motion in space is given. The curve along with the velocity vector at the time $t = \dfrac{3\pi}{2}$ is illustrated.

The velocity and acceleration vectors at time *t* are
$v(t) = r'(t) = -3\sin t\,\hat{i} + 3\cos t\,\hat{j} - 4\cos t\sin t\,\hat{k}.$

$\quad = -3\sin t\,\hat{i} + 3\cos t\,\hat{j} - 2\sin 2t\,\hat{k}.$

$a(t) = r''(t) = -3\cos t\,\hat{i} - 3\sin t\,\hat{j} - 4\cos 2t\,\hat{k}.$

The speed is

$|v(t)| = \sqrt{(-3\sin t)^2 + (3\cos t)^2 + (-2\sin 2t)^2}$.

When $t = \dfrac{3\pi}{2}$, we have

$v\left(\dfrac{3\pi}{2}\right) = 3\hat{i}$, $a\left(\dfrac{3\pi}{2}\right) = 3\hat{j} + 4k$, $|v(t)| = 3$.

The graph of the velocity vector can be formed by using Matlab code.

```
t=0:.1:3*pi/2;
subplot (1,1,1)
plot3(3.*cos(t),3.*sin(t),2.*(cos(t).^2),'--g*')
xlabel('x')
ylabel('y')
zlabel('z')
```
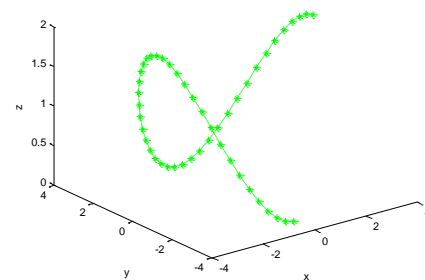

Fig 2 The Curve and the Velocity Vector when

$t = \dfrac{3\pi}{2}$ for the Motion.

4. APPLICATIONS OF PROJECTILE MOTIONS

The equations for projectile motion, suppose that the projectile behaves like a particle moving in a vertical coordinate plane and that the only force acting on the projectile during its flight is constant force of gravity, which always points straight down. We assume that the projectile is launched from the origin at time $t = 0$ into the first quadrant with an initial velocity v_0. If v_0 makes an angle α with the horizontal, then $v_0 = (|v_0|\cos\alpha)\hat{i} + (|v_0|\sin\alpha)\hat{j}$.

If we use the simpler notation v_0, for the initial speed, then

$\quad v_0 = (v_0\cos\alpha)\hat{i} + (v_0\sin\alpha)\hat{j}$.

(7)

The projectile's initial position is

$\quad x_0 = 0\,\hat{i} + 0\,\hat{j} = 0$.   (8)

By Newton's second law of motion, the force acting on the projectile is equal to the projectile's mass m times

its acceleration, or $m\left(\dfrac{d^2 r}{d t^2}\right)$ if r is the projectile's

position vector and t is time. If the force $-mg\,\hat{j}$, then

$$m\frac{d^2 r}{d t^2}=-mg\,\hat{j} \qquad \text{and} \qquad \frac{d^2 r}{d t^2}=-g\,\hat{j}\,,$$ where g is

the acceleration due to gravity.
Ideal Projectile Motion Equation is

$$r =\left(v_0\cos\alpha\right)t\,\hat{i}+\left(\left(v_0\sin\alpha\right)t-\frac{1}{2}gt^2\right)\hat{j}. \qquad (9)$$

Equation (9) is the vector equation for ideal projectile motion. The angle α is the projectile's launch angle (firing angle, angle of elevation), and $v_{0,}$ as we said before, is the projectile's initial speed.

4.1. Example

An archer fires an arrow at an angle of $30^{.}$ above the horizontal with an initial speed of $900\dfrac{\text{m}}{\text{sec}}$. We can find the distance of an arrow will be 20 sec later reached.

$v_0 = 900$, $\alpha = 30^{.}$, $g = 9.8$, $t = 20^{.}$

The vector equation for ideal projectile motion is

$$r =\left(v_0\cos\alpha\right)t\,\hat{i}+\left(\left(v_0\sin\alpha\right)t-\frac{1}{2}gt^2\right)\hat{j}$$

$$\approx 15588.46\,\hat{i}+7040\,\hat{j}.$$

Twenty seconds after firing, the arrow is 7040 m above ground.
The graph of the distance of an arrow can be formed by using Matlab code.

```
x0=0;
y0=0;
k=0;
angle=30*(pi./180);
v=900;
g=9.8;
t=0:0.1:20;
x=x0+v*cos(angle)*t;
y=y0+v*sin(angle)*t-(g*t.^2)/2;
figure
plot (x,y,'--g*')
xlabel('Distance');
ylabel('Height');
```
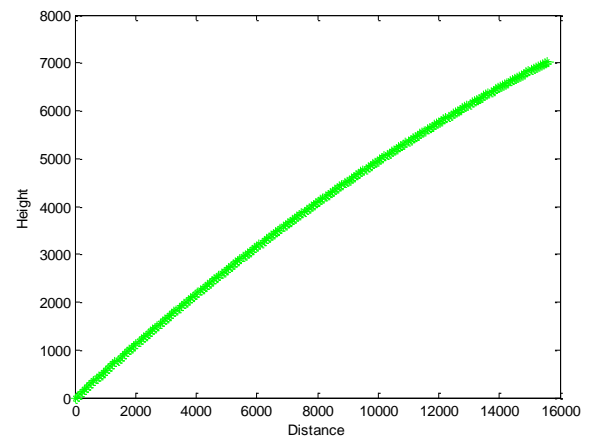

Fig 3 The Distance of an Arrow.

4.2. Definitions

For ideal projectile motion when an object is launched from the origin over a horizontal surface with initial speed v_0 and launch angle α :

Maximum height: $y_{\max}=\dfrac{\left(v_0\sin\alpha\right)^2}{2g}$. $\qquad (10)$

Flight time: $\qquad t=\dfrac{2v_0\sin\alpha}{g}$. $\qquad (11)$

Range: $\qquad R=\dfrac{v_0{}^2}{g}\sin 2\alpha$. $\qquad (12)$

4.3. Example

A golfball is hit when it is 6 ft above the ground. It leaves the bat with initial speed of $102\dfrac{\text{ft}}{\text{sec}}$, making an angle of $30^{.}$ with the horizontal. At the instant the ball is hit, an instantaneous gust of wind blows in the horizontal direction directly opposite the direction the ball is taking toward the outfield, adding a component of $-6.6\hat{i}\left(\dfrac{\text{ft}}{\text{sec}}\right)$ to the ball's initial velocity . A vector equation (position vector) for the path of the golfball can be expressed. The height of golfball and when does it reaches maximum height can be found. Assuming that the ball is not placed, its range and flight time can be calculated.

The initial velocity is

$$v_0 =\left(v_0\cos\alpha\right)\hat{i}+\left(v_0\sin\alpha\right)\hat{j}-6.6\hat{i}$$
$$=\left(102\cos 30^{.}-6.6\right)\hat{i}+\left(102\sin 30^{.}\right)\hat{j}.$$

The initial position is $r_0 = 0\,\hat{i}+6\,\hat{j}$.

$$\frac{d^2 r}{d t^2}=-g\,\hat{j}\;.$$

Integrating both sides, $\dfrac{d\,\boldsymbol{r}}{d\,t} = -\left(gt\right)\hat{j} + \boldsymbol{v}_0$.

The projectile's position vector is

$$\boldsymbol{r} = -\frac{1}{2}gt^2\,\hat{j} + \boldsymbol{v}_0 t + \boldsymbol{r}_0.$$

$$\boldsymbol{r} = 16t^2\,\hat{j} + \left(102\cos 30^\circ - 6.6\right)t\,\hat{i} + \left[\left(102\sin 30^\circ\right)t + 6\right]\hat{j}.$$

The golfball reaches its highest point when the vertical component of velocity is $\dfrac{dy}{dt} = 102\sin 30^\circ - 32t = 0$.

$$t = \frac{102\sin 30^\circ}{32} \approx 1.594 \text{ sec} .$$

The maximum height is

$$y_{\max} = 6 + \left(102\sin 30^\circ\right)\left(1.594\right) - 16\left(1.594\right)^2 \approx 46.641 \text{ ft.}$$

That is, the maximum height of the golfball is about 46.641 ft reached about 1.594 sec after leaving the bat.

For time, $\qquad 6 + \left(102\sin 30^\circ\right)t - 16t^2 = 0$

$$t = 3.301\ (or)t = -0.114 .$$

Therefore, $t = 3.301$ sec.

The horizontal range is $R = \left(102\cos 30^\circ - 6.6\right)\left(3.301\right)$

$$R \approx 269.81 \text{ ft} .$$

Thus, the horizontal range is about 2.9681 ft and the flight time is about 3.301 sec.

The graph of the horizontal range of a golfball can be formed by using Matlab code.

```
x0=-6.6;
y0=6;
k=0;
angle=30*(pi./180);
v=102;
g=32;
t1=0:0.1:1.594;
x=x0+v*cos(angle)*t1;
y=y0+v*sin(angle)*t1-(g*t1.^2)/2;
figure
plot (x,y,'--g*');
hold on;
t2=0:0.1:3.301;
x=x0+v*cos(angle)*t2;
y=y0+v*sin(angle)*t2-(g*t2.^2)/2;
figure
plot (x,y,'--g*');
hold off;
xlabel ('Maximum Height');
```

ylabel ('Horizontial Range');

Fig 4 The Horizontial Range of a Golfball.

4.4. Example

In an artillery, a cannon is fired at a speed of $840\,\dfrac{\text{m}}{\sec}$ at an angle of 60°. The time of it reaches 21 km downrange can be computed.

The distance downrange is $x = \left(\boldsymbol{v}_0 \cos\alpha\right)t$.

$$\left(21\text{km}\right)\left(\frac{1000\,\text{m}}{1\text{km}}\right) = \left(840\,\frac{\text{m}}{\text{s}}\right)\left(\cos 60^\circ\right)t .$$

$$t = 50 \text{ sec} .$$

This problem can be solved by using Matlab code.

```
clc,clear
D=input('Enter initial speed, angle, downrange with[]:');
v0=D(1); angle=D(2); x0=D(3);
t = x0/(v0*cos(pi*(angle/180)));
fprintf('t= %.1fs\n',t);
```

The result is t= 50.0 sec.

The graph of the time of a cannon can be formed by using Matlab code.

```
function [x, y] = trajectory (a, time)
x0=0;
y0=0;
k=0;
angle=60*(pi./180);
v=840;
g=9.8;
t=0:0.1:50;
x=x0+v*cos(angle)*t;
y=y0+v*sin(angle)*t-(g*t.^2)/2;
figure
plot (x,y,'--g*')
xlabel('Distance');
ylabel('Height');
end
```
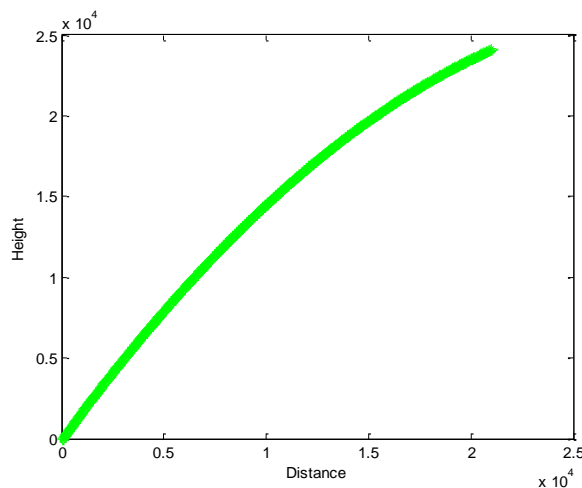
Fig 5 The Height of a Cannon.

4.5. Example

A projectile is fired with an initial speed of $500\,\dfrac{m}{sec}$ at an angle of elevation of $45°$, time and distance of the projectile strike can be found, the height of the projectile can be computed when it is 5 km downrange and the greatest height reached by the projectile can be calculated.

The time is $\quad t = \dfrac{2\boldsymbol{v}_0}{g}\sin\alpha$.

$$t = \frac{2\left(500\,\dfrac{m}{s}\right)\left(\sin 45°\right)}{9.8\,\dfrac{m}{s^2}} \approx 72.2 \text{ sec}.$$

The horizontal range is $\quad R = \dfrac{\boldsymbol{v}_0{}^2}{g}\sin 2\alpha$.

$$R = \frac{\left(500\,\dfrac{m}{s}\right)^2}{9.8\,\dfrac{m}{s^2}}\left(\sin 90°\right) \approx 25510.2 \text{ m}.$$

The distance downrange is $\quad x = \left(\boldsymbol{v}_0\cos\alpha\right)t$.

$$5000\,m = \left(500\,\frac{m}{s}\right)\left(\cos 45°\right)t$$

$$t = \frac{5000\,m}{\left(500\,\dfrac{m}{s}\right)\left(\cos 45°\right)} \approx 14.14 \text{ sec}.$$

For height, $y = \left(\boldsymbol{v}_0\sin\alpha\right)t - \dfrac{1}{2}gt^2$.

$$y = \left(500\,\frac{m}{s}\right)\left(\sin 45°\right)(14.14) - \frac{1}{2}\left(9.8\,\frac{m}{s^2}\right)(14.14)^2$$

$$y \approx 4020 \text{ m}.$$

For greatest height, $\quad y_{max} = \dfrac{\left(\boldsymbol{v}_0\sin\alpha\right)^2}{2g}$.

$$y_{max} = \frac{\left[\left(500\,\dfrac{m}{s}\right)\left(\sin 45°\right)\right]^2}{2\left(9.8\,\dfrac{m}{s^2}\right)} \approx 6378 \text{ m}.$$

This problem can be illustrated and calculated by using Matlab code.

Inputs:
```
clc, clear
D=input ('Enter initial speed, angle, downrange, gravitional force with []:');
v0=D(1); theta=D(2); x0=D(3); g=D(4);
t_flight = 2*v0*(sin(pi*(theta/180)))/g;
R = v0^2*(sin(2*pi*(theta/180)))/g;
t = x0/(v0*cos(pi*(theta/180)));
y = (v0*(sin(pi*(theta/180))))*t-(g*t^2)/2;
y_max = (v0*(sin(pi*(theta/180))))^2/(2*g);
fprintf ('The time is %0.1fs\n',t_flight);
fprintf ('The horizontial range is %0.1fm\n',R);
fprintf ('The height is %0.1fm\n',y);
fprintf ('The greatest height is %0.1fm\n',y_max);
t=linspace (0,t_flight,30);
xdot0=v0*cos(pi*(theta/180));
ydot0=v0*sin(pi*(theta/180));
y0=0;
x=xdot0*t+x0;
y=-(g/2)*t.^2+ydot0*t+y0;
plot(x,y,'--g*');
axis equal
xlabel (' Horizontal range ');
ylabel (' Greatest height ');
```
Outputs:
The time is 72.2 sec.
The horizontial range is 25510.2 m.
The height is 4020.0 m.
The greatest height is 6377.6 m.
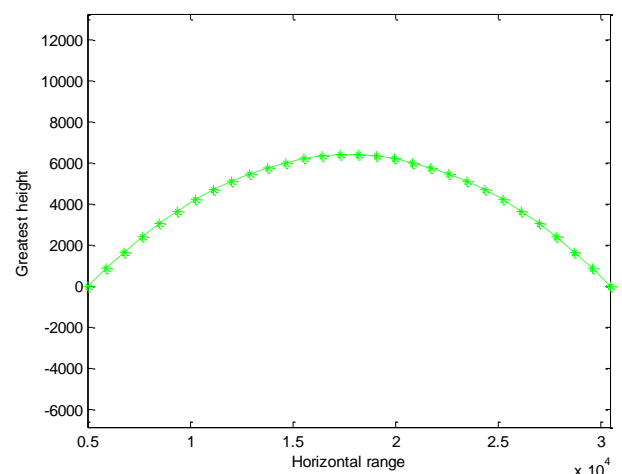
Fig 6 The Greatest Height of a Projectile.

4.6. Example

A baseball is thrown from the stands 32 ft above the field at an angle of 30° up from the horizontial. Time and the distance of the ball will strike the ground can be found if its initial speed is $32\frac{ft}{sec}$.

The height of the projectile at time $t \geq 0$ is

$$y = y_0 + (v_0 \sin\alpha)t - \frac{1}{2}gt^2.$$

$$y = 32\,ft + \left(32\frac{ft}{sec}\right)(\sin 30°)t - \frac{1}{2}\left(32\frac{ft}{sec^2}\right)t^2$$

$$y = 32 + 16t - 16t^2.$$

The ball hits the ground when y=0, $32 + 16t - 16t^2 = 0$

$t = -1 (or) t = 2$.

Therefore, $t = 2$ sec.

The distance downrange is $x = (v_0 \cos\alpha)t$.

$$x = \left(32\frac{ft}{sec}\right)(\cos 30°)(2) \approx 55.4\,ft.$$

The graph of the time and distance of a ball can be formed by using Matlab code.

```
function [x,y] = trajectory (a,time);
x0=0;
y0=32;
k=0;
angle=30*(pi./180);
v=32;
g=32;
t=0:0.1:2;
x=x0+v*cos(angle)*t;
y=y0+v*sin(angle)*t-(g*t.^2)/2;
figure
plot (x,y,'--g*')
xlabel('Distance');
ylabel('Height');
end
```
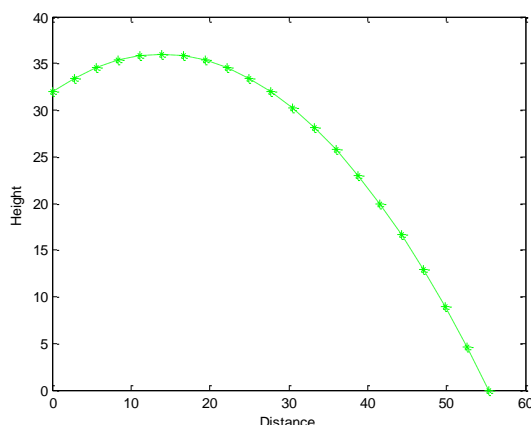

Fig 7 The Distance of a Ball.

5. CONCLUSIONS

We have presented the applications of projectile motion in space. Projectile motion describes how an object fired at some angle from an initial position, moves in a vertical coordinate plane. It can be widely used in golf and baseball sports. The muzzle speed of a gun can be calculated. Mathematics is a part of real life. It is evident in the modern technological devices we use in every day experiences and objects around us. Although, mathematics is understood to be only useful in the classroom. Mathematics can also be applied to one the most popular activities on the canon, golfball, basketball. The motion in space and the vector-valued functions with their many important applications are applied in engineering, sports and economics.

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Numeric Solutions of Non-Homogeneous Ordinary Differential Equations by Runge-Kutta Method and Runge-Kutta-Fehlberg Method with Matlab Codes

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ABSTRACT*:* In this paper, the Runge-Kutta method and Runge-Kutta Fehlberg method are ideal for solving nonlinear nonhomogeneous ordinary differential equations (ODEs) in difference of compute the approximate solutions to more elementary methods. We will describe Runge-Kutta-Fehlberg method of error estimate. We will then proceed to explain what steps the method actually carries out in solving the nonlinear nonhomogeneous ODEs along with the MATLAB codes. We used to write the Runge-Kutta method and Runge-Kutta-Fehlberg method solver and the output of the code, given some basic nonlinear nonhomogeneous ODEs. We will then end by discussing the pitfalls of this way of solving nonlinear nonhomogeneous ODEs.

KEYWORDS*: error estimate, MATLAB, ordinary differential equations, Runge-Kutta, Runge-Kutta Fehlberg.*

1. INTRODUCTION

Numerical methods are used to find the numerical approximations to the solutions of ODEs. Many differential equations cannot be solved using symbolic computation. We consider the numerical method of Runge-Kutta and Runge-Kutta-Fehlberg methods for solving the first-order and higher-order nonlinear nonhomogeneous ODEs [Erwin Kreyszig (2011)].

We also discussed accuracy analysis of numerical solutions of initial value problems for ODEs. The simplest numerical method of Euler method is extended using the Runge-Kutta method. We will study is numeric solution of error estimate by using Runge-Kutta method and Runge-Kutta-Fehlberg method with MATLAB [Brian H. Hahn (2010)], [Jaan Kiusalaas (2005)]. The calculation is also written in the ten decimals place.

2. NUMERIC SOLUTIONS OF ODEs

The main initial idea is that we can obtain approximations to the solution of such an ODEs at points that area a distance *h* apart by using the first two terms of Taylor's formula from calculus. We use these approximations to construct the iteration formula for a method known as Euler's method. While this method is rather unstable and of little practical use, it serves as a pedagogical tool and a starting point toward understanding more sophisticated methods such as the Runge-Kutta method and its variant the Runge-Kutta-Fehlberg (RKF) method, which are popular and useful in practice. As it is usual in mathematics, one tends to generalize mathematical ideas [Erwin Kreyszig (2011)].

2.1 Higher-Order ODEs

ODE is of nth order if the nth derivative of the unknown function is the highest occurring derivative. Thus, the ODE is of the form:

$$F(x, y, y', \cdots, y^{(n)}) = 0 \qquad (1)$$

where lower order derivatives and *y* itself may or may not occur. An initial value problem for the ODE consists of *n* initial conditions

$$y(x_0) = \mathbf{K}_0,$$

$$y'(x_0) = \mathbf{K}_1, \cdots, y^{(n-1)}(x_0) = \mathbf{K}_{n-1}, \qquad (2)$$

with given in the open interval I considered, and given $\mathbf{K}_0, \cdots, \mathbf{K}_{n-1}$.

2.2 Fourth-Order Runge-Kutta (RK4 Method)

The fourth-order Runge-Kutta method is obtained from the Taylor series along the same lines as the second-order method. Since the derivation is rather long and not very instructive, we skip it. The final form of the integration formula again depends on the choice of the parameters; that is, there is no unique Runge-Kutta foruth-order formula [Islam, Md.A (2015)]. The most popular version, which is known simply as the Runge-Kutta method, entails the following sequence of operations:

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4). \quad (3)$$

2.3 Runge-Kutta-Fehlberg (RKF or RK5 Method)

E. Fehlberg proposed and developed error control by using two RK methods of different orders to go from (x_n, y_n) to (x_{n+1}, y_{n+1}). The difference of the computed *y*-values at x_{n+1} gives an error estimate to be used for step size control. Fehlberg discovered two RK formulas that together need only six function evaluations per step. We present these formulas here because RKF has become quite popular [Erwin Kreyszig (2011)].

$$k_1 = hf(x_n, y_n),$$

$$k_2 = hf\left(x_n + \frac{1}{4}h, y_n + \frac{1}{4}k_1\right),$$

$$k_3 = hf\left(x_n + \frac{3}{8}h, y_n + \frac{3}{32}k_1 + \frac{9}{32}k_2\right),$$

$$k_4 = hf\left(x_n + \frac{12}{13}h, y_n + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3\right),$$

$$k_5 = hf\left(x_n + h, y_n + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right),$$

$$k_6 = hf\left(x_n + \frac{1}{2}h, y_n - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4\right.$$
$$\left. - \frac{11}{40}k_5\right),$$

$$y_{n+1} = y_n + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6, \quad (4)$$

$$y_{n+1}^* = y_n + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4104}k_4 - \frac{1}{5}k_5. \quad (5)$$

2.4 Error Estimate of Runge-Kutta-Fehlberg Method

$$\varepsilon_{n+1} \approx y_{n+1} - y_{n+1}^* \quad (6)$$

We can solve the error estimate to be used in Runge-Kutta-Fehlberg method [Ogunrinde, R.B (2012)]. Let us illustrate these facts with a simple example.

3. NUMERIC SOLUTIONS OF NON-HOMOGENEOUS ODEs WITH MATLAB CODES

This section describes how to calculate the approximate solutions of Runge-Kutta method and Runge-Kutta-Fehlberg method in given examples using matlab codes.

3.1 Example: First Order Non-Homogeneous ODEs

Apply the Runge-Kutta and Runge-Kutta-Fehlberg methods to the initial value problem in $y' = (y - x - 1)^2 + 2, \quad y(0) = 1$ choosing $h = 0.1$, as

before and computing exact solution and error estimate in 10 steps.

The exact solution $y = \tan x + x + 1$ is obtained using the initial condition [See Table 1, Column 4].

3.2 Example: Higher Order Non-Homogeneous ODEs

Apply the Runge-Kutta and Runge-Kutta-Fehlberg methods to the initial value problem in $y'' + 11y' + 10y = 10x + 11, \quad y(0) = 2, \quad y'(0) = -10$ choosing $h = 0.2$, as before and computing exact solution and error estimate in 10 steps.

The exact solution $y = e^{-x} + e^{-10x} + x$ is obtained using the initial condition [See Table 3, Column 4].

3.3 Steps for MATLAB implementation

The purpose of using an example is to show the details of implementing the typical steps of Runge-Kutta method, so that it will be clear exactly what computations are being executed. For some reasons, MATLAB does not include Runge-Kutta functions. However, MATLAB has very sophisticated ones using Runge-Kutta-Fehlberg algorithms [Jaan Kiusalaas (2005)]. We will show how to use one of them in this section.

3.3.1 Create user-defined function files: The MATLAB mscript **runKut4.m** can be used to calculate approximate solutions of ODEs [May Zun (2019)]. The MATLAB mscript **printSol.m** can be used to print *x* solution and *y* solution arrays in tabular form at [May Zun (2019)].

3.3.2 Create user-defined function file: Example_3.1.m

```
function F=Example_3.1 (x,y)
% Differential.eqs.used in Example3.1
F=(y-x-1)^2+2;
```

3.3.3 Create user-defined function file: Example_3.2.m

```
function F=Example_3.2 (x,y)
% Differential.eqs.used in Example3.2
F=zeros(1,2);
F(1)=y(2);
F(2)=-11*y(2)-10*y(1)+10*x+11;
```

3.3.4 Create user-defined function file:runKutFeh.m.

```
%runKutFeh.m
function [xSol,ySol] = runKutFeh(dEqs,x,y,xStop,h)
```

% 6th-order Runge--Kutta integration.
% USAGE: [xSol,ySol] = runKut4(dEqs,x,y,xStop,h)
% INPUT:
% dEqs = handle of function that specifies the
% 1st-order differential equations
% F(x,y) = [dy1/dx dy2/dx dy3/dx ...].
% x,y = initial values; y must be row vector.
% xStop = terminal value of x.
% h = increment of x used in integration.
% OUTPUT:
% xSol = x-values at which solution is computed.
% ySol = values of y corresponding to the x-values.
if size(y,1) > 1; y = y'; end % y must be row vector
xSol = zeros(2,1); ySol = zeros(2,length(y));
xSol(1) = x; ySol(1,:) = y;
i = 1;
while x < xStop
   i = i + 1;
   h = min(h,xStop - x);
   K1 = h*feval(dEqs,x,y);
   K2 = h*feval(dEqs,x +h/4,y + (1/4)*K1);
   K3 = h*feval(dEqs,x +(3/8)*h,y
+(3/32)*K1+(9/32)*K2);
   K4 = h*feval(dEqs,x +(12/13)*h,y+(1932/2197)*K1-
(7200/2197)*K2+(7296/2197)*K3);
   K5 = h*feval(dEqs,x +h,y+(439/216)*K1-
8*K2+(3680/513)*K3-(845/4104)*K4);
   K6 = h*feval(dEqs,x +0.5*h,y-(8/27)*K1+2*K2-
(3544/2565)*K3+(1859/4104)*K4-(11/40)*K5);
   y = y + ((16/135)*K1 + (6656/12825)*K3
+(28561/56430)*K4-(9/50)*K5+(2/55)*K6);
   x = x + h;
   xSol(i) = x; ySol(i,:) = y; % Store current soln.
end

4. RESULTS FOR RUNGE-KUTTA AND RUNGE-KUTTA-FEHLBERG METHODS

In Table 1, the data indicate the Example 3.1 of approximate solutions $y_{1,n}$ in RK4 method and RK5 method with exact values by using MATLAB codes.
Using **runKut4.m, printSol.m** and **Example_3.1.m script** together in common window:

>> [x,y]=runKut4(@Example_3.1,0,1,1,0.1);
>> printSol(x,y,1).

We get the tabular output, which displays the approximate solutions of our initial value problem, as shown in [Table 1, Column 2].

Similarly, **runKutFeh.m** script is also used in common window:>>
[x,y]=runKutFeh(@Example_3.1,0,1,1,0.1);
>> printSol(x,y,1).
We get the approximate solutions of our initial value problem, as shown in [Table 1, Column 3].

For [Table 1, Column 4], exact solution is using Matlab code in common window:
>> x=0:.1:1;
>> y=tan(x)+x+1;
>> fprintf('%14.10f',y).

Table 1: Approximate Solutions and Exact Values of Example 3.1

x_n	y_n		
	RK4	**RK5**	**Exact**
0.0	1.0000000000	1.0000000000	1.0000000000
0.1	1.2003345891	1.2003346725	1.2003346721
0.2	1.4027098782	1.4027100374	1.4027100355
0.3	1.6093360393	1.6093362546	1.6093362496
0.4	1.8227929929	1.8227932298	1.8227932187
0.5	2.0463023076	2.0463025124	2.0463024898
0.6	2.2841367567	2.2841368531	2.2841368083
0.7	2.5422885694	2.5422884689	2.5422883805
0.8	2.8296390611	2.8296387346	2.8296385571
0.9	3.1601587828	3.1601585865	3.1601582176
1.0	3.5574064428	3.5574085377	3.5574077247
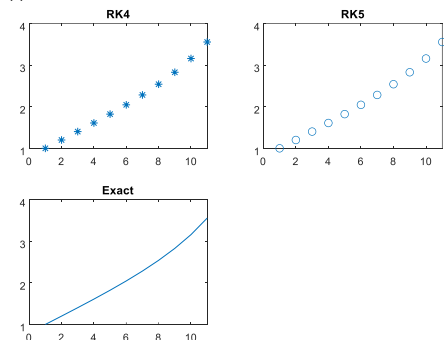
Fig 1. Approximate solutions of Runge-Kutta method and Runge-Kutta-Fehlberg method with exact values for Example 3.1

In Table 2, the data indicate the Example 3.1 of error estimate in RK5 by using (6).

Table 2: Error Estimate and Actual Error of RK5 for Example 3.1

x_n	y_n	Error	
	RK5 y_n^*	**Error Estimate**	**Actual Error**
0.0	1.0000000000	0.0000000000	0.0000000000
0.1	1.2003346695	0.0000000030	-0.0000000004
0.2	1.4027100306	0.0000000068	-0.0000000019
0.3	1.6093362415	0.0000000131	-0.0000000050
0.4	1.8227932062	0.0000000236	-0.0000000111
0.5	2.0463024733	0.0000000391	-0.0000000226
0.6	2.2841367945	0.0000000586	-0.0000000448
0.7	2.5422883957	0.0000000732	-0.0000000884
0.8	2.8296386852	0.0000000494	-0.0000001775

| 0.9 | 3.1601587156 | -0.0000001291 | -0.0000003689 |
| 1.0 | 3.5574094084 | -0.0000008707 | -0.0000008130 |

In Table 3, the data indicate the Example 3.2 of approximate solutions $y_{1,n}$ in RK4 method and RK5 method with exact values by using MATLAB codes. Using **runKut4.m, printSol.m** and **Example_3.2.m** script together in common window:

>> [x,y]= runKut4(@Example_3.2,0,[2 -10],2,0.2);
>> printSol(x,y,1).

We get the tabular output, which displays the approximate solutions of our initial value problem, as shown in [Table 3, Column 2].
Similarly, **runKutFeh.m** script is also used in common window:

>> [x,y]=runKutFeh(@Example_3.2,0,[2 -10],2,0.2);
>> printSol(x,y,1).

We get the approximate solutions of our initial value problem, as shown in [Table 3, Column 3]. For [Table 3, Column 4], using Matlab code in common window:

>> x=0:.2:2; >> y=exp(-x)+exp(-10*x)+x;
>> fprintf('%14.10f',y).

Table 3: Approximate Solutions and Exact Values of Example 3.2

x_n	y_n		
	RK4	RK5	Exact
0.0	2.0000000000	2.0000000000	2.0000000000
0.2	1.3520666667	1.1161665949	1.1540660363
0.4	1.1814353822	1.0798137090	1.0886356849
0.6	1.1858538619	1.1497365567	1.1512903883
0.8	1.2616803075	1.2494189733	1.2496644267
1.0	1.3720004645	1.3678880982	1.3679248411
1.2	1.5025716494	1.5011949448	1.5012003561
1.4	1.6470596515	1.6465969300	1.6465977955
1.6	1.8020540241	1.8018964163	1.8018966305
1.8	1.9653543820	1.9652987879	1.9652989035
2.0	2.1353564835	2.1353351913	2.1353352853
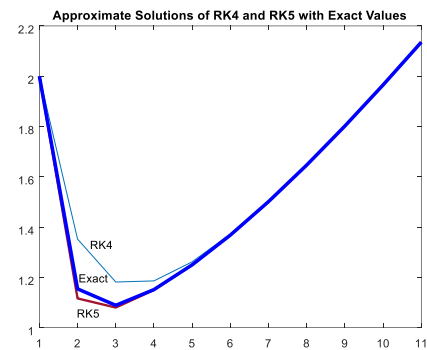
Fig 2. Approximate solutions of Runge-Kutta method and Runge-Kutta-Fehlberg method with exact values for Example 3.2

In Table 4, the data indicate the Example 3.2 of error estimate in RK5 by using (6).

Table 4: Error Estimate and Actual Error of RK5 for Example 3.2

x_n	y_n	Error	
	RK5 y_n^*	Error Estimate	Actual Error
0.0	2.0000000000	0.0000000000	0.0000000000
0.2	1.0443712821	0.0717953128	0.0378994414
0.4	1.0709766950	0.0088370140	0.0088219759
0.6	1.1488274953	0.0009090614	0.0015538316
0.8	1.2493283061	0.0000906672	0.0002454534
1.0	1.3678783364	0.0000097618	0.0000367429
1.2	1.5011931159	0.0000018289	0.0000054113
1.4	1.6465959168	0.0000010132	0.0000008655
1.6	1.8018955382	0.0000008781	0.0000002142
1.8	1.9652979857	0.0000008022	0.0000001156
2.0	2.1353344623	0.0000007290	0.0000000940

5. CONCLUSIONS

On the basis of the above discussion we get the result obtained by analytical methods is always providing accurate solution but numerical solution always providing approximate result. To validate the applicability of the proposed method, nonlinear nonhomogeneous ODEs examples have been considered and solve for different value of *x*. We observed from numerical solution presented in [Table 1 and 3], Runge-Kutta-Fehlberg method approximate and exact solutions very well. The accuracy of present method is better than the other existing methods. Thus, Runge-Kutta-Fehlberg method is more accurate and a preferable method to find the approximate solution of the fourth order initial value problems. Furthermore,

Runge-Kutta-Fehlberg method is decreased the solutions of error. These two methods could be quickly and easily resolved with MATLAB codes [See Fig.1 and Fig.2]. The approximate values of Example 3.1 are $y_{10} = 3.5574064428$ of RK method and $y_{10} = 3.5574085377$ of RKF [See Table 1]. And then, approximate values of Example 3.2 are $y_{10} = 2.1353564835$ of RK4 method and $y_{10} = 2.1353351913$ of RKF [See Table 3]. Moreover, we illustrated the error estimate of Runge-Kutta-Fehlberg method $\varepsilon_{10} = -0.0000008707$ of Example 3.1 [See Table 2] and $\varepsilon_{10} = 0.0000007290$ of Example 3.2 in [See Table 4]. In the RKF method, the error estimate is slightly error more than the actual error.

ACKNOWLEDGEMENT

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Solving the Forcing Functions in Electric Circuit by Using Fourier Series

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ABSTRACT*:* In this paper, Fourier series and their engineering use from a practical point of view with the approximation of periodic functions are discussed. Trigonometric form of Fourier series with even and odd functions are explained. The Fourier coefficients of half-wave rectifier in any period are found. Moreover, the force response for the nth harmonic by working in the frequency domain can be calculated. Some applications are expressed to illustrate how practical problems.

KEYWORDS: *Fourier series, Fourier coefficients, sinusoidal voltage, forcing function.*

1. INTRODUCTION

Fourier series are the basic appliance for representing periodic functions, which play an important role in applications. A Fourier series is an infinite expansion of function in terms sines and cosines. We will discuss the trigonometric form of the Fourier series for the function $f(t)$, with the process of determining the values of the constants a_0, a_n and b_n [Kreyszig (2017)]. We will calculate the Fourier series of a sinusoidal voltage through a half-wave rectifier that clips the negative portion of the wave and illustrate the "half-sinusoidal" wave form [William (1978), Steven (2001)].

2. TRIGONOMETRIC FORM OF FOURIER SERIES

Fourier series are infinite series that represent periodic functions in terms of cosines and sine. As such, Fourier series are of greatest importance to the engineer and applied mathematician. To define Fourier series, we first need some background material. A function $f(x)$ is called a periodic function if $f(x)$ is defined for all real x, except possibly at some points, and if there is some positive number p, called a period of $f(x)$, such that $f(x + p) = f(x)$ for all x [Kreyszig (2017)].
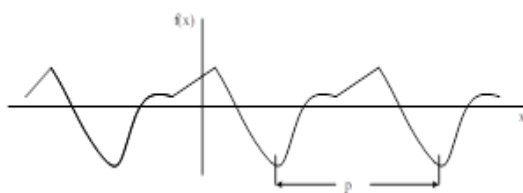
Fig. 1. Periodic function of period p

the function $f(x)$ of a period $p = 2L$, the Fourier series is

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi}{L} + b_n \sin\frac{n\pi}{L}) \qquad (1)$$

with the Fourier coefficients

$$\left. \begin{array}{l} (a)\ a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx \\ (b)\ a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi}{L}x\ dx, n=1,2,3,\cdots \\ (c)\ b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi}{L}x\ dx, n=1,2,3,\cdots \end{array} \right\} \qquad (2)$$

2.1 Simplifications of even and odd functions

If $f(x)$ is an even function, that is $f(-x) = f(x)$ in figure 2, its Fourier series (1) reduces to a Fourier Cosine Series.
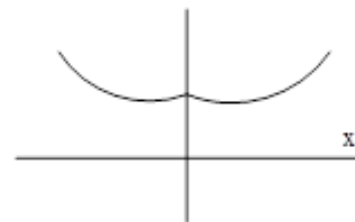
Fig. 2. Even function

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{L}x \quad (f\ even) \qquad (3)$$

with coefficients

$$\left. \begin{array}{l} a_0 = \frac{1}{L}\int_{0}^{l} f(x)dx \\ a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi}{L}x\ dx, n=1,2,3,\cdots \end{array} \right\} \qquad (4)$$

If $f(x)$ is an odd function, that is $f(-x) = -f(x)$ in figure 3, its Fourier series (1) reduces to a Fourier Sine Series.
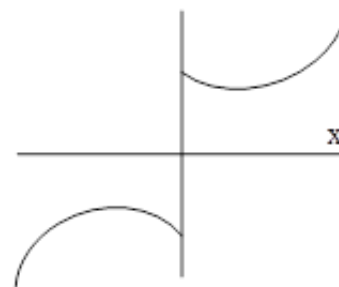
Fig. 3. Odd Function

$$f(x) = a_0 + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{L}x \quad (f\ odd) \qquad (5)$$

with coefficients

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi}{L}x\,dx, \quad n = 1,2,3,\cdots \qquad (6)$$

2.2 Half-wave rectifier

A sinusoidal voltage $E\sin\omega t$, where t is time is passed through a half-wave rectifier in figure 4 that clips the negative portion of the wave the Fourier series of resulting periodic function

$$u(t) = \begin{cases} 0 & if\ -L < t < 0 \\ E\sin\omega t & if\ 0 < t < L \end{cases},$$

$p = 2L = 2\pi/\omega$, $L = \pi/\omega$ can be calculated.
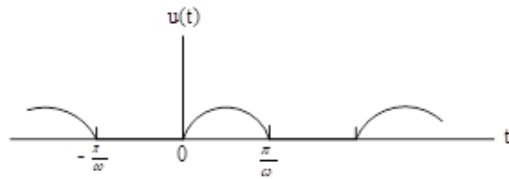
Fig 4. Half-wave rectifier

The coefficients of Fourier series are

$$a_0 = \frac{1}{2L}\int_{-L}^{L} u(t)\,dt = E/\pi \quad \text{and}$$

$$a_n = \omega/\pi \int_0^{\frac{\pi}{\omega}} E\sin\omega t\cos n\omega t\,dt,$$

if $n = 1$, then $a_1 = 0$.

If n=2,3,…, then $a_n = \begin{cases} 0, & n\ is\ odd \\ -\frac{2E}{(n-1)(n+1)\pi}, & n\ is\ even \end{cases}$.

$$b_n = \frac{\omega}{\pi}\int_0^{\frac{\pi}{\omega}} E\sin\omega t\sin n\omega t\,dt,$$

if $n = 1$, then $b_1 = \frac{E}{2}$.

If $n = 2,3,\cdots$ then $b_n = 0$.

The series of half-wave rectifier is

$$u(t) = \frac{E}{\pi} + \frac{E}{2}\sin\omega t$$

$$-\frac{2E}{\pi}\left(\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \cdots\right).$$

3. EVALUATION OF FOURIER COEFFICIENTS IN CIRCUIT

We first consider a periodic function $f(t)$, defined by the functional relationship $f(t) = f(t + T)$ where T is the period. The Fourier Series in (1) states that $f(t)$ may be represented by the infinite series

$$f(t) = a_0 + a_1\cos\omega_0 t + a_2\cos 2\omega_0 t + \cdots$$
$$+ b_1\sin\omega_0 t + b_2\sin 2\omega_0 t + \cdots$$

$$= a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n\sin n\omega_0 t),$$

where the fundamental frequency ω_0 is related to the period T by

$$\omega_0 = \frac{2\pi}{T} = 2\pi f_0$$

$$a_0 = \frac{1}{\pi}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)dt$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos n\omega_0 t\,dt$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin n\omega_0 t\,dt$$

and where a_0, a_n and b_n are constants that depend upon n and $f(t)$[William (1978)].

3.1 The "half-sinusoidal" wave

The "half-sinusoidal" wave form shown in figure 5 represents the voltage response obtained at the output of a half-wave rectifier circuit, a nonlinear circuit whose purpose is to convert a sinusoidal input voltage to a (pulsating) approximation to dc. The Fourier series representation of this waveform can be found.
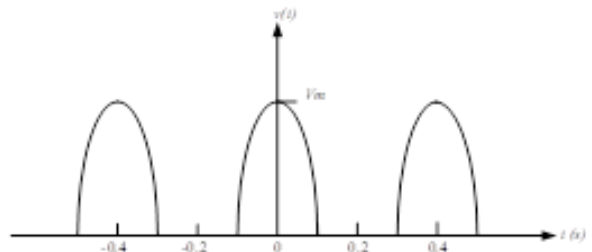
Fig. 5. The output of a half-wave rectifier

From the figure 5, the period is seen to be $T = 0.4s$ and thus $f_0 = 2.5\ Hz$ and $\omega_0 = 5\pi$ rad/s.

$$v(t) = \begin{cases} 0, & -0.2 \le t \le -0.1 \\ V_m\cos 5\pi t, & -0.1 \le t \le 0.1 \\ 0, & 0.1 \le t \le 0.2 \end{cases}$$

Since $v(t)$ is even function, $b_n = 0$,

$$a_0 = \frac{2}{T}\int_0^{\frac{T}{2}} v(t)dt = \frac{V_m}{\pi} \quad,$$

$$a_n = \frac{4}{\pi}\int_0^{\frac{T}{2}} v(t)\cos n\omega_0 t\,dt, \quad n = 1,2,\ldots \quad,$$

if $n = 1$, then $a_1 = \frac{V_m}{2}$.

If $n = 2,3,\ldots,$

then $a_n = 5V_m\left[\frac{\sin\frac{(1-n)\pi}{2}}{(1-n)5\pi} - \frac{\sin\frac{(1+n)\pi}{2}}{(1=n)5\pi}\right]$.

Thus the Fourier series is

$$v(t) = a_0 + \sum_{n-1}^{\infty} a_n\cos\frac{n\pi}{L}t$$

$$= \frac{V_m}{\pi} + \frac{V_m}{2}\cos 5\pi t + \frac{2V_m}{3\pi}\cos 10\pi t$$

$$-\frac{2V_m}{15\pi}\cos 20\pi t + \frac{2V_m}{35\pi}\cos 30\pi t - + \cdots$$

3.2 Voltage waveform

The Fourier series for the voltage waveform as shown in figure 6 can be written.
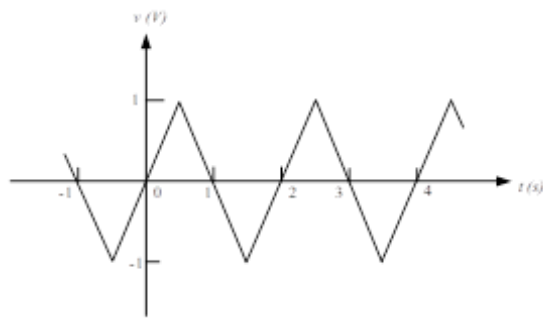
Fig.6. Voltage waveform

From the figure 6, we have

$$v(t) = \begin{cases} -2t - 2, -1 \leq t \leq -\frac{1}{2} \\ 2t \quad , -\frac{1}{2} \leq t \leq \frac{1}{2} \\ -2t + 2, \frac{1}{2} \leq t \leq \frac{3}{2} \\ 2t - 4, \frac{3}{2} \leq t \leq \frac{5}{2} \end{cases},$$

$T=2, f_0 = \frac{1}{T} = \frac{1}{2} = 0.5$ Hz, $\omega_0 = 2\pi f_0 = \pi$ rad/s.

Since $v(t)$ is odd function, $a_0 = a_n = 0$ and

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} v(t) \sin n\omega_0 t \, dt, \quad n = 1, 2, \dots$$

$$= \frac{2}{2} \int_{-1}^{1} v(t) \sin n\pi t \, dt$$

$$= \frac{8}{n^2 \pi^2} [\sin \frac{n\pi}{2} - \cos n\pi].$$

Therefore the Fourier series is

$$v(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$= \frac{16}{\pi^2} \left(\sin \pi t + \frac{1}{5^2} \sin 5\pi t + \frac{1}{9^2} \sin 9\pi t + \cdots \right)$$

$$- \frac{8}{\pi^2} \left(\frac{1}{2^2} \sin 2\pi t + \frac{1}{4^2} \sin 4\pi t + \frac{1}{6^2} \sin 6\pi t + \cdots \right).$$

4. SOLVING THE PERIODIC FORCING FUNCTIONS

We may now explain an arbitrary periodic forcing function as the sum of an infinite number of sinusoidal forcing functions by using the Fourier series. The forced response to each of these functions may be determined by conventional steady-state analysis, and the form of the natural response may be determined from the poles of an appropriate network transfer function.

4.1 Example

The circuit in figure7(a), can be determined the periodic response $i(t)$ corresponding to the forcing function shown in figure 7(b) if $i(0) = 0$.
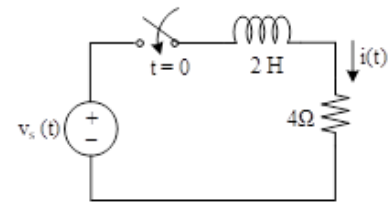
(a)
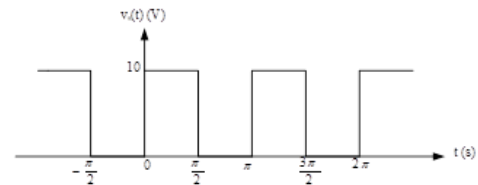
(b)

Fig.7. (a) A simple series RL circuit subjected to a periodic forcing function $v_s(t)$.(b) The form of the forcing function.

$T = \pi$ sec, $f_0 = \frac{1}{T} = \frac{1}{\pi}$ Hz, $\omega_0 = 2\pi f_0 = 2$rad/sec

$$a_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \, dt = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} 10 \, dt = 5,$$

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos \omega_0 nt \, dt$$

$$= \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} 10 \cos 2nt \, dt = 0, \text{ and}$$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin \omega_0 nt \, dt$$

$$= \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} 10 \sin 2nt \, dt = \begin{cases} 0, & n \text{ is even} \\ \frac{20}{n\pi}, & n \text{ is odd} \end{cases}.$$

$$v_s(t) = a_o + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$v_s(t) = 5 + \frac{20}{\pi} \sum_{n=1(\text{odd})}^{\infty} \frac{\sin 2nt}{n}$$

$$v_s(t) = v_{s_1}(t) + v_{s_2}(t)$$

The response to the dc component is

$$i_{f_1}(t) = \frac{V_{s_1}}{R} = \frac{5}{4} = 1.25$$

$$v_{s_2}(t) = \frac{20}{n\pi} \sum_{n=1(\text{odd})}^{\infty} \sin 2nt = \frac{20}{n\pi} \angle - 90°.$$

The impedance offered by the RL circuit at this frequency is

$$\mathbf{Z}_T = R + j\omega L = 4 + j(2n)2 = 4 + j4n \text{ and}$$

the component of the forced response at this frequency is

$$i_{f_2}(t) = \frac{v_{s_2}(t)}{\mathbf{Z}_T}$$

$$= \frac{5}{n\pi} \left(\frac{1}{\sqrt{1+n^2}} \right) \cos(2nt - 90° - \tan^{-1} n)$$

$$= \frac{5}{\pi(1+n^2)} \sum_{n=1(\text{odd})}^{\infty} \left(\frac{\sin 2nt}{n} - \cos 2nt \right).$$

$i_f(t) = i_{f_1}(t) + i_{f_2}(t)$

$i_f(t) = 1.25 + \frac{5}{\pi}\sum_{n=1(odd)}^{\infty}\left[\frac{\sin 2nt}{n(1+n^2)} - \frac{\cos 2nt}{1+n^2}\right].$

The natural response of this simple circuit is

$i_n(t) = Ae^{-\frac{Rt}{L}}$

$\quad = Ae^{-2t}.$

The complete response can be written

$i(t) = i_f(t) + i_n(t)$

$\quad = Ae^{-2t} + 1.25 + \frac{5}{\pi}\sum_{n=1(odd)}^{\infty}\left[\frac{\sin 2nt}{n(1+n^2)} - \frac{\cos 2nt}{1+n^2}\right].$

Letting $t = 0$, we can find A using $i(0^+) = 0$.

$A = -1.25 + \frac{5}{\pi}\sum_{n=1(odd)}^{\infty}\frac{1}{1+n^2},$

$A = 1.25 + \frac{5}{\pi}(0.720) = -0.104.$

$i(t) = i_n(t) + i_f(t) = -0.104e^{-2t} + 1.25$

$\qquad + \frac{5}{\pi}\sum_{(n=1)odd}^{\infty}\left[\frac{\sin 2nt}{n(1+n^2)} - \frac{\cos 2nt}{1+n^2}\right].$

4.2 Example

For the circuit of figure 8(a), can be calculated $v(t)$ if $i_s(t)$ is given by figure 8(b) and $v(0) = 0$.

(a)

(b)
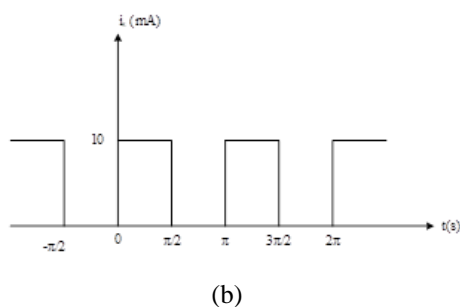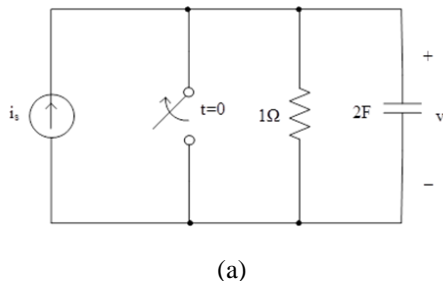
Fig. 8. (a) A simple series RC circuit subjected to a periodic forcing function i_s.(b) The form of the forcing function.

$T = \pi$ sec, $f_0 = \frac{1}{T} = \frac{1}{\pi}$ Hz, $\omega_0 = 2\pi f_0 = 2$rad/sec

By Example 4.1, the Fourier series expansion for the current is

$i_s(t) = 5 + \frac{20}{\pi}\sum_{n=1(odd)}^{\infty}\frac{\sin 2nt}{n}.$

The force response to the nth harmonic in the frequency domain is

$i_{s_2} = \frac{20}{\pi n}\sum_{n=1(odd)}^{\infty}\sin 2nt = \frac{20}{n\pi}\angle -90°.$

The equation for the capacitance of the nth harmonic frequency is

$X_C = -\frac{j}{n\omega_0 C} = -\frac{j}{4n}.$

The current at the capacitor is

$i_c = i_{s_2}\left(\frac{R}{R+X_C}\right) = \frac{80\angle -90°}{\pi(4n-j)}.$

The forced response of the $v(t)$ at the nth harmonic in the frequency domain is

$v_2(t) = i_{s_2}X_C$

$\quad = \frac{20\angle -90°}{\pi n(1+4nj)}$

$\quad = \frac{20}{n\pi}\left(\frac{1}{\sqrt{1+16n^2}}\right)\cos(2nt - 90° - \tan^{-1} 4n)$

$\quad = \frac{20}{\pi}\sum_{n=1(odd)}^{\infty}\left[\frac{\sin 2nt}{n(1++16n^2)} - \frac{\cos 2nt}{(1+16n^2)}\right].$

The response to the dc component is $v_1(t) = i_{s_1}R = 5.$

$v_f(t) = v_1(t) + v_2(t)$

$\quad = 5 + \frac{20}{\pi}\sum_{n=1(odd)}^{\infty}\left[\frac{\sin 2nt}{n(1++16n^2)} - \frac{\cos 2nt}{(1+16n^2)}\right]$

The equation for the natural response is

$v_n(t) = V_0e^{-\frac{t}{RC}} = V_0e^{-\frac{t}{2}}.$

The complete response of the system is the addition the natural response and the forced response. Therefore

$v(t) = v_f(t) + v_n(t)$

$\quad = 5 + \frac{20}{\pi}\sum_{n=1(odd)}^{\infty}\left[\frac{\sin 2nt}{n(1++16n^2)} - \frac{\cos 2nt}{(1+16n^2)}\right]$

$\qquad + V_0e^{-\frac{t}{2}}.$

From $v(0) = 0$, we can find the value of V_0.

$V_0 = -5 + \sum_{n=1(odd)}^{\infty}\frac{80}{\pi(1+16n^2)} = -3.2108.$

Thus

$v(t) = 5 + \frac{20}{\pi}\sum_{n=1(odd)}^{\infty}\left[\frac{\sin 2nt}{n(1++16n^2)} - \frac{\cos 2nt}{(1+16n^2)}\right]$

$\qquad -3.1208\, e^{-\frac{t}{2}}.$

5. CONCLUSIONS

There are many applications of Fourier series in engineering science. Among these, we have presented finding the series of rectifier wave form, sinusoidal wave form and voltage wave form. We concluded that, the force response of electrical circuits in the field of

electrical engineering will be more effective if the Fourier series of engineering mathematics are studied.

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Application of Prim's Algorithm Method for Travelling Nine Pagodas Named as Shin Koe Shin in Dawei Region

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ABSTRACT: A minimum spanning tree is subset of a graph that connects all vertices with minimum possible total edge weights. The minimum spanning tree problem is an important application in network design. The aim of the present study is to find minimum weight spanning tree using Prim's algorithm of weighted graph. Therefore, the connected ways from pagoda to pagoda in Dawei are computed by using the construction of minimum spanning tree in this research. According to the research, Shin Koe Shin pagodas in Dawei are assumed as vertices and the major roads that link these pagodas are edges. And then, in order to use Prim's algorithm, Shin Dat Wae Pagoda is supposed as the starting point. The result points out that the total shortest distance from Shin Dat Wae Pagoda to other eight pagodas is 203.5 Kilometers after calculating with Prim's algorithm.

 KEYWORDS: *Weighted- graph, connected, minimum spanning tree & Prim's Algorithm*

1. INTRODUCTION

In many fields, the applications of graph theory play important role for various modeling problems in real life such as travelling, transportation, traffic control, communications, and various computer applications and so on. It is observed that social applications begin with some basic graph terminology. Graph is used to represent the network, where the vertices can represent cities, stations, computer etc., and the edges of the graph can represent roads, link safety and so on and the goals can be cost output performance.

Prim's algorithm is observed in 1930 by mathematicians, Vojtech Jarnik and Robert C. Prim, is greedy algorithm that finds a minimum spanning tree for a connected weighted graph. It discovers a tree of a connected weighted graph, which includes every vertex and the total weight of all the edges in the tree is less than or equal to every possible spanning tree.

We, then, discuss some important concepts in graph theorem, including paths and cycles. Tree form of the most widely is used subclasses of graphs. Computer science, in particular, makes extensive use of trees. Spanning tree is basically used to find a minimum path to connect all vertices in a graph. It can be used Prim's algorithm on a network to find a minimum spanning tree. When we visit a website like Google Maps or use our smart phone to ask for driving directions, we are usually looking for a shortest path between the two

locations. These computer applications are used to represent the street maps as graphs, and estimate driving times as edged weights.

2. TERMINOLOGY

Graph theory is a branch of mathematics which is used to model, analyze and solve many real-life problems. We need to be familiar with some basic terminology used in graph theory.

2.1 Definitions

A **graph** consists of two finite sets, a set *V* of nodes, called vertices, and a set *E* of connecting lines, called edges, such that each edge connects two vertices, called the endpoints of the edge. We write *G = (V, E)*. A **subgraph** is part of a graph. A **path** is a finite sequence of edges, such that the end vertex of one edge in the sequence is the start vertex of the next, and in which no vertex appears more than once. A **cycle** is a closed path, i.e. the end vertex of the last edge is the start vertex of the first edge. Two vertices are connected if there is a path between them. A **graph is connected** if all its vertices are connected. A **weighted graph** is a graph in which each edge has a weight (some real number).**Weight of a graph** is the sum of the weights of all edges. A connected cyclic graph is called a **tree**. In other words, a connected graph with no cyclic is called a tree. A tree with "n" vertices has "n-1"dges. **A spanning tree** of a graph is a subgraph that includes all the vertices of *G*. A connected graph *G* can have more than one spanning tree. All possible spanning tree of graph *G*, have the same number of edges and vertices. In a weighted graph, **a minimum spanning tree** is a spanning tree that has minimum weight than all other spanning tree of the same graph [Bondy (1976)].

Now, it is presented the following example with the concepts of basic graph theory.

2.2 Example

The weighted graph *G* of Fig 1 .(a) shows five cities and the costs of building roads between certain pairs of cities. It is desired to build the lowest-cost roads system that will connect the five cities. The solution can be represented by a subgraph. This subgraph must be a spanning tree since it must contain all the vertices (so that each city is in the road system), it must be connected (so that any city can be reached from any other), and it must have a unique simple path between each pair of vertices (since a graph containing

multiple paths between a vertex pair could not represent a minimum-cost system). Thus what is needed is a spanning tree the sum of whose weights is a minimum. Such a tree is called a minimum spanning tree [Johnsongaugh (2005)].
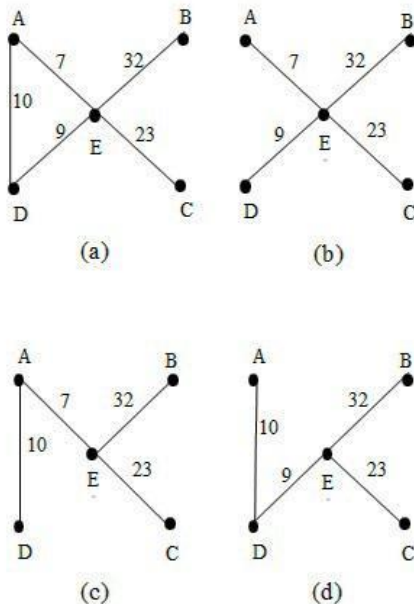
Fig1. (a) Five cities and the costs of building roads between certain pairs of them, (b) A spanning tree of weight 71, (c) A spanning tree of weight 72, (d) A spanning tree of weight 74,

Trees shown in Fig 1. (c) and (d) are spanning trees of Fig 1. (a). The weights of the trees are 72 and 74. These trees are not a minimal spanning tree since spanning tree shown in Fig 1. (b) has weight 71. It will see later that tree in Fig 1. (b) of the minimal spanning tree for Fig 1.(a).

3. PRIM'S ALGORITHM AND APPLICATION

The algorithm to find a minimal spanning tree that we will discuss is known as Prim's Algorithm. This algorithm builds a tree by iteratively adding edges until a minimal spanning tree is obtained. The algorithm begins with a single vertex. Then at each iterations, it adds to the current tree a minimum-weight edge that does not complete a cycle [Johnsonbaugh (2005)].

3.1 Prim's Algorithm

This algorithm finds a minimum spanning tree in a connected, weighted graph.
Input: A connected, weighted graph with vertices 1,…, n.
Output: The set of edge E in a minimum spanning tree.
*Step*1: Initialize the minimum spanning tree with a single vertex, randomly chosen from the graph.
*Step*2: Select an edge that connects the tree with a vertex not yet in the tree, so that the weight of the edge is minimal and inclusion of the edge does not form a cycle.

*Step*3: Add the selected edge and the vertex that it connects to the tree.

*Step*4: Repeat step 2 and 3 until we get a minimum spanning tree.

3.2. Analysis of Data

Dawei region is located in the upper third of Tanintharyi Region which is situated in southern most Myanmar. And also Dawei City is the capital of Thanintharyi Region. It lies on the northern bank of Dawei River and it is 614.3km (381.7miles) apart from Yangon. It is 30km (18.6 miles) from the Andaman Sea. It is bounded with Ye Township in the north, Thailand in the south and Adman Sea in the west. It is composed of four townships namely, Dawei, Launglon, Yebyu and Thayetchaung Townships. The total area of it is 5407.09 square miles. Dawei is beautiful with Mountains, Forests, Beaches, etc. It is famous for 9 historical pagodas called Shin Koe Shin. Shin Koe Shin pagodas can be divided into three types which are mountain pagodas, forest pagodas and town pagodas. Three mountain pagodas are Shin Sandaw Pagoda (Kaleinaung Su Taung Ptae Pagoda), Shin Maw Taung Pagoda and Shin Taunk Pone Pagoda. Three forest pagodas are Shin Dat Wae Pagoda, Shin Zalun Pagoda and Shin Tauk Htein Pagoda and the third is three town pagodas are Shin Pin Khayu Pagoda, Shin Oak Aww Pagoda and Shin Mothtee Pagoda.

Su Taung Ptae Shin Sandaw Pagoda was built on Kaleinaung Hill in Yebyu Townships. The Hill is 47 miles north of Dawei. Three pieces of relic hair of Kakusanda, Konagamana and Gotama Buddhas who had enlightened in Buddha World were enshired in the rocky reliquary. We claimed to the top by car and enjoyed the beauty of the scenery of the countryside. Shin Maw Taung Pagoda is a hilly pagoda in launglon Township, located in Kyat Hlut Village in which is more than 52 miles from Dawei. After driving for about 3 miles to the top of Dawei Promontory, we reached the pagoda. The road to Shin Maw Taung Pagoda is close to the seashore so we can enjoy the beautiful shoreline, mangrove forests and the life style of residents along the ways. Shin Taunk Pone Pagoda built on more than 2000 feet high hill 18 miles from Dawei, Thayetchaung Township. Even thorough the mountain was steeply, we could reach the pop by car where the pagoda was situated. From the peak of mountain, I could see the zigzag mountains and natural beauty of the countryside.

Shin Dat Wae Pagoda built by Shin Zan is located in a Maung Mae Shaung Village about 7 miles from Dawei Yayphyu road at the north of Dawei. Emblems of Srivaccha were expressed on concrete floral works created around the upper part of bell at Shin Zalun Pagoda, located in old Thar Gara City in Launglon Township. Shin Pin Khayu Pagoda located on Arzarni Road in south Kayat Pyin Ward of Dawei was built about 300 year ago.

Shin Oak Aww Pagoda which is inside the campus of Shwe Kaung Monastery, an ancient wooden

monastic building at Painneltaw Ward in Dawei. Shin Mothtee Pagoda was assumed that Shin Mothtee Village about 6 miles from Dawei, derived from Mottathu nakhagara City. It located on right side of Dawei-Myeik motor road, was enshrined with air Shin Mothtee Buddha image.Shin Koe Shin pagodas, which are famous across Myanmar, do not have firmly historical evidences [Wikipedia (2018)].

Using the information described in the table, this table values represent distance (km) for the respective routes form Google Map. The connected, weighted graph of Fig 3 shows nine pagodas and the costs of building roads between certain pair of pagodas. It is desired to find the cost effective road system that will connected the nine pagodas. It can be found the cost-effective of building. Since the number of vertices in Fig 3 is 9, the number of edges in a spanning tree in Fig 4 is 8. Consider the following graph in Fig 3 and shows the various steps involved in the construction of the minimum spanning tree. By Prim's algorithm, we "8" start with the vertex and proceed [Abhilasha (2013)].

Fig 2. Photograph of Dawei Region

Table1. shows   distance (km) values from one pagoda to one pagoda in Dawei.

  1. Shin Pin Khayu

  2. Shin Mothtee

  3. Shin Taunk Pone

  4. Shin maw Taung

  5. Shin Tauk Htein

  6. Shin Oak Aww

  7. Shin Zalun

  8. Shin Dat Wae

  9. Shin San Daw

Table1. The distance between Shin Koe Shin pagodas using kilometer

	1	2	3	4	5	6	7	8	9
1	-	9.3	-	-	-	1.5	-	12	-
2	9.3	-	19	-	-	-	-	20	-
3	-	19	-	-	-	-	-	-	-
4	-	-	-	-	86.5	89.5	95.5	98.5	-
5	-	-	-	86.5	-	4.8	9.4	14	-
6	1.5	-	-	89.5	4.8	-	14	13	-
7	-	-	-	95.5	9.4	14	-	22	61
8	12	20	-	98.5	14	13	22	-	61
9	-	-	-	-	-	-	61	61	-

It can be shown how Prim's algorithm finds a minimum spanning tree for the graph in Fig 3.

Thus, it denoted that nine pagodas as vertices and it are assumed that the start vertex is 8 (Shin Dat Wae Pagoda).

We add vertex 8 to the minimum spanning tree.

Edge	Weight
(8, 1)	12
(8, 2)	20
(8, 4)	98.5
(8, 5)	14
(8, 6)	13
(8, 7)	22
(8, 9)	61

The edge (8, 1) with minimum weight is selected. The vertex 1 is added to the minimum spanning tree and edge (8, 1) is added to *E*.

Edge	Weight
(8, 2)	20
(8, 4)	98.5
(8, 5)	14
(8, 6)	13
(8, 7)	22
(8, 9)	61
(1, 2)	9.3
(1, 6)	1.5

The edge (1, 6) with minimum weight is selected. The vertex 6 is added to the minimum spanning tree and edge (1, 6) is added to *E*.

Edge	Weight
(8, 2)	20
(8, 4)	98.5
(8, 5)	14
(8, 7)	22
(8, 9)	61
(1, 2)	9.3
(6, 4)	89.5
(6, 5)	4.8
(6, 7)	14

The edge (6, 5) with minimum weight is selected. The vertex 5 is added to the minimum spanning tree and edge (6, 5) is added to *E*.

Edge	Weight
(8, 2)	20
(8, 4)	98.5
(8, 7)	22
(8, 9)	61
(1, 2)	9.3
(6, 4)	89.5
(6, 7)	14
(5, 4)	86.5
(5, 7)	9.4

The edge (1, 2) with minimum weight is selected. The vertex 2 is added to the minimum spanning tree and edge (1, 2) is added to *E*.

Edge	Weight
(8, 4)	98.5
(8, 7)	22
(8, 9)	61
(6, 4)	89.5
(6, 7)	14
(5, 4)	86.5
(5, 7)	9.4
(2, 3)	19

The edge (5, 7) with minimum weight is selected. The vertex 7 is added to the minimum spanning tree and edge (5, 7) is added to *E*.

Edge	Weight
(8, 4)	98.5
(8, 9)	61
(6, 4)	89.5
(5, 4)	86.5
(2, 3)	19
(7, 4)	95.5
(7, 9)	61

The edge (2, 3) with minimum weight is selected. The vertex 3 is added to the minimum spanning tree and edge (2, 3) is added to *E*.

Edge	Weight
(8, 4)	98.5
(8, 9)	61
(6, 4)	89.5
(5, 4)	86.5
(7, 4)	95.5
(7, 9)	61

The edge (8, 9) with minimum weight is selected. The vertex 9 is added to the minimum spanning tree and edge (8, 9) is added to *E*.

Edge	Weight
(8, 4)	98.5
(5, 4)	86.5
(6, 4)	89.5
(7, 4)	95.5

The edge (5, 4) with minimum weight is selected. The vertex 4 is added to the minimum spanning tree and edge (5, 4) is added to *E*. The minimal spanning tree constructed is shown in Fig 4.
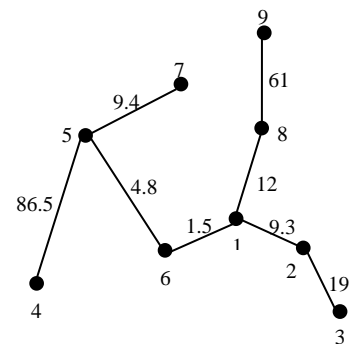
Fig 4. A spanning tree of
weight 203.5 of the graph of
the Fig 3.

By applying Prim's algorithm to the network in Fig 4, the minimum spanning tree or shortest possible road network 203.5 kilometer is obtained.

4. CONCLUSIONS

The minimum spanning tree problem is an important application in network design. It is noted that when the shortest path of Shin Koe Shin pagodas in Dawei is calculated by the minimum spanning tree using Prim's algorithm, the minimum route is 203.5 kilometers. Hence the shortest path of the minimum spanning tree is (8, 1), (1, 6), (6, 5), (1, 2), (5, 7), (2, 3), (8, 9), (5, 4). It is hope that the present study will be beneficial for visitors who want get information to go to Shin Koe Shin pagodas in Dawei with the shortest path.

By using Prim's algorithm applied in this paper, the shortest path of pagodas and resorts not only for road transportation but also for marine transportation in any cities can be computed.

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Solving the Bounded Regions with Double Integrals in Real Life

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ABSTRACT: In this paper, Fubini's Theorem for calculating double integrals is expressed. Double integrals over rectangles and planes can be calculated as iterated integrals. The practical problems of the areas and average values of the bounded, nonrectangular regions are solved. The population density of a certain bacterium on the xy-plane and the planar region on Earth are calculated. Double integrals are also computed as iterated integrals, with the main practical problem being that of determining the limits of integration. The areas of the leaf of the rose, cardioid and snail shell are discussed by using double integrals. The area and the volume of the cylinder that lies inside the cardioid and outside the circle can be evaluated by using double integrals. The average height of the paraboloid over the square can be evaluated by using double integrals.

KEYWORDS: *area, bounded regions, double integral, population density, volume.*

1. INTRODUCTION

There are many different methods for solving problems. Among these, we express the practical problems for real life by using the double integrals. We define and evaluate double integrals over bounded regions in the plane which are more general than rectangles. These double integrals are also evaluated as iterated integrals, with the main practical problem being that of determining the limits of integration. In section 2, Fubini's Theorem for evaluating double integrals is stated with bounded regions. Moreover, we show how to use double integrals to calculate the areas of bounded regions in the plane and to find the average value of a function of two variables. Integrals are sometimes easier to evaluate if we change to polar coordinates, in section 3. Then, we show how to accomplish the change and how to evaluate integrals over regions whose boundaries are given by polar equations. In section 4, we compute the volumes of the bounded regions by double integrals [Thomas (1914)]. The Figures are illustrated and the problems are solved by MATLAB code [Palm (1944)].

2. AREAS AND AVERAGE VALUES OF THE BOUNDED REGIONS

Fubini's Theorem said that double integrals over rectangular and nonrectangular regions can solve a double integral by integrating with respect to one or two variables at a time. Let $f(x, y)$ be continuous on a region R. If R is defined by $a \le x \le b$, $g_1(x) \le y \le g_2(x)$, with g_1 and g_2 continuous on $[a, b]$, then

$$\iint_R f(x, y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y)\, dy\, dx. \qquad (1)$$

If R is defined by $c \le y \le d$, $h_1(y) \le x \le h_2(y)$, with h_1 and h_2 continuous on $[c, d]$, then

$$\iint_R f(x, y)\, dA = \int_c^d \int_{h_1(x)}^{h_2(x)} f(x, y)\, dx\, dy. \qquad (2)$$

We can evaluate the areas of bounded regions in the plane by using double integrals. The **area** of a closed, bounded plane region R is $A = \iint_R dA.$

2.1 The total population of bacteria within the rectangle

If $f(x, y) = (10{,}000 e^y) / \left(1 + \dfrac{|x|}{2}\right)$ is the 'population density' of a certain bacterium on the xy-plane, where x and y are measured in centimeters, then we can find the total population of bacteria within the rectangle $-5 \le x \le 5$ and $-2 \le y \le 0$.

The graph of the 'population density' of a certain bacterium on the xy-plane can be expressed by using MATLAB code.

```
clc, clear, close all;
[x, y] = meshgrid (-5:.1:5, -2:.1:0);
z=10000*exp(y)/(1+abs(x)/2);
surf(z);
xlabel ('x');
ylabel ('y');
zlabel ('z');
title ('Total population');
```
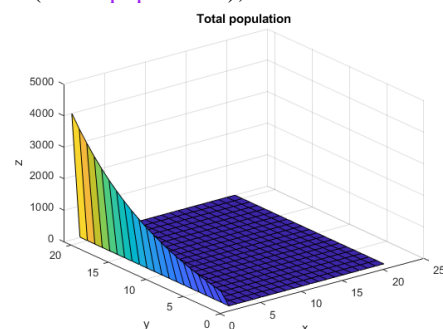

Fig 1 The 'Population Density' of a Certain Bacterium on the xy-plane.

The total population of bacteria is

$$\int_{-5}^{5}\int_{2}^{0}\frac{10,000e^{y}}{1+\left|x\right|/2}\,dy\,dx = 10,000\int_{-5}^{5}\int_{2}^{0}\frac{e^{y}}{1+\left|x\right|/2}\,dy\,dx$$

$$= 10,000\int_{-5}^{5}\frac{1}{1+\left|x\right|/2}\left[e^{y}\right]_{-2}^{0}dx = 10,000\int_{-5}^{5}\frac{\left(e^{0}-e^{-2}\right)}{1+\left|x\right|/2}dx$$

$$\approx 43,329 \,.$$

This can be calculated by MATLAB code [Lopez (2014)].

```
clc, clear;
fun=@ (x, y) (10000.*exp(y))./(1+(abs(x)./2));
q=integral2(fun, -5,5, -2,0);
fprintf ('Total population of bacteria is %.2f \n', q)
Total population of bacteria is 43328.80.
```

2.2 The population density of a planar region on Earth

Suppose that $f(x,y)=100(y+1)$ represents the population density of a planar region on Earth, where x and y are measured in miles. We can find the number of people in the region bounded by the curves $x=y^2$ and $x=2y-y^2$.

The graph of the 'population density' of a planar region on Earth can be illustrated by using MATLAB code.

```
clc, clear, close all;
syms x y
x1=y.^2; x2=2. * y-y.^2;
fplot (x1, [0,1]); hold on;
fplot (x2, [0,1]); hold off;
title ('region bounded by x=y.^2 and x=2. * y-y.^2');
```
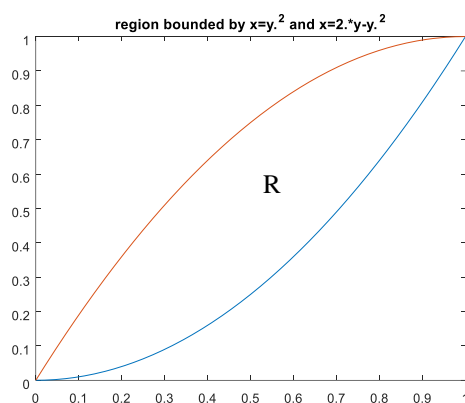

Fig 2 The Population Density of a Planar Region on Earth.

For any y between 0 and 1, x may vary from $x=y^2$ to $x=2y-y^2$.

The number of people in the region is

$$\int_{0}^{1}\int_{y^2}^{2y-y^2}100(y+1)dx\,dy = \int_{0}^{1}\left[100(y+1)x\right]_{x=y^2}^{x=2y-y^2}dy$$

$$= \int_{0}^{1}100(y+1)\left(2y-2y^2\right)dy = 50 \,.$$

This can be expressed by MATLAB code.

```
clc,clear;
syms x y
fun=@ (x, y)100.*(y+1);
A=int (int (fun, y.^2, 2.*y-y.^2),y,0,1);
fprintf ('The number of people in the region is %d \n', A)
The number of people in the region is 50.
```

2.3 The average height of the paraboloid

The average height of the paraboloid $z=x^2+y^2$ over the square $0 \le x \le 2$, $0 \le y \le 2$ can be evaluated.

The graph of the average height of the paraboloid can be illustrated by using MATLAB code.

```
clc, clear, close all;
[x, y] = meshgrid (0:.1:2,0:.1:2);
z=x.^2+y.^2;
surf(z);
xlabel ('x');
ylabel ('y');
zlabel ('z');
title ('Average Height');
```
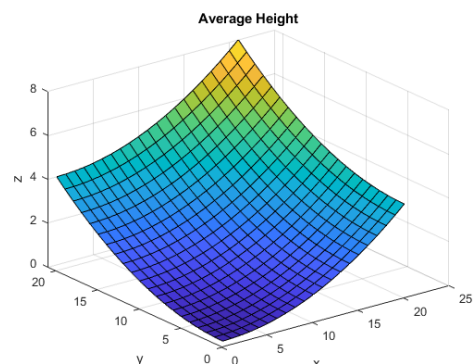

Fig 3 The Average Height of the Paraboloid $z=x^2+y^2$.

The average height is $\dfrac{1}{area\,of\,R}\iint\limits_{R}f\,dA$

$$= \frac{1}{4}\int_{0}^{2}\int_{0}^{2}\left(x^2+y^2\right)dy\,dx$$

$$= \frac{1}{4}\int_{0}^{2}(2x^2+\frac{8}{3})dx = \frac{8}{3}\approx 2.67 \,.$$

This can be calculated by MATLAB code.

```
clc, clear;
fun=@ (x, y) x.^2+y.^2;
q=integral2(fun,0,2,0,2);
A= (1. / 4) * q;
```

fprintf ('The Average Height is %.2f \n', A)
The Average Height is 2.67.

3. CHANGING CARTESIAN INTEGRALS INTO POLAR INTEGRALS

To change a Cartesian integral $\iint\limits_R f(x,y)\,dx\,dy$ into a polar integral, use substitute $x = r\cos\theta$ and $x = r\sin\theta$, and replace $dx\,dy$ by $r\,dr\,d\theta$ in the Cartesian integral. Then supply polar limits of integration for the boundary of R [Kreyszig (1999)]. The Cartesian integral becomes

$$\iint\limits_R f(x,y)\,dxdy = \iint\limits_G f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta.$$

(3)

Where G denotes the same region of integration now described in polar coordinates.

3.1 The area of the Cardioid

The area of the Cardioid overlapping a circle that lies inside the Cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$ can be illustrated.

The graph of the area of inside the Cardioid and outside the circle can be expressed by using MATLAB code.

```
clc, clear, close all;
t=0:2*pi/100:2*pi;
r=1+cos(t);
figure;
polar (t, r, 'b');
hold on;
plot(sin(t), cos(t),'r')
axis square;
```
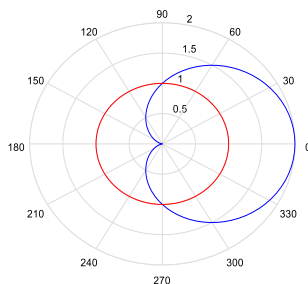

Fig 4 The Area between inside the Cardioid and outside the Circle.

The shaded area is

$$2\int_0^{\frac{\pi}{2}}\int_1^{1+\cos\theta} r\,dr\,d\theta = 2\int_0^{\frac{\pi}{2}}\left[\frac{r^2}{2}\right]_{r=1}^{r=1+\cos\theta} d\theta$$

$$= \int_0^{\frac{\pi}{2}}\left[(1+\cos\theta)^2 - (1)^2\right]d\theta = \int_0^{\frac{\pi}{2}}\left(2\cos\theta + \cos^2\theta\right)d\theta$$

$$= \int_0^{\frac{\pi}{2}}\left(2\cos\theta + \frac{1+\cos\theta}{2}\right)d\theta = 2 + \frac{\pi}{4} \approx 2.79.$$

This can be expressed by MATLAB code.
```
clc, clear;
syms theta r
polarfun = @ (theta, r) r;
q=int (int (polarfun, r, 1, 1+cos(theta)), theta, 0, pi./2);
A=2.*q;
fprintf ('The shaded area is %.2f \n', A)
```
The shaded area is 2.79.

3.2 The area enclosed of one leaf of the rose

The area enclosed by one leaf of the rose $r = 12\cos 3\theta$ can be found.

The graph of the area enclosed by one leaf of the rose can be illustrated by using MATLAB code.

```
clc, clear, close all;
t = 0:2*pi/100:2*pi;
r = 12*cos(3*t);
figure; polar (t, r,' r');
```
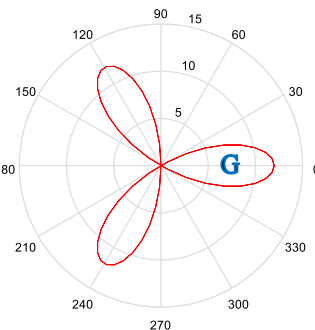

Fig 5 The Area of One Leaf of the Rose.

The shaded area is $2\int_0^{\frac{\pi}{6}}\int_0^{12\cos 3\theta} r\,dr\,d\theta = 2\int_0^{\frac{\pi}{6}}\left[\frac{r^2}{2}\right]_{r=0}^{r=12\cos 3\theta} d\theta$

$$= \int_0^{\frac{\pi}{6}}(12\cos 3\theta)^2\,d\theta = 144\int_0^{\frac{\pi}{6}}\frac{1+\cos 6\theta}{2}\,d\theta = 72\int_0^{\frac{\pi}{6}}(1+\cos 6\theta)\,d\theta$$

$$= 12\pi \approx 37.70.$$

This can be calculated by MATLAB code.
```
clc, clear;
syms theta r
polarfun=@ (theta, r) r;
q=int (int (polarfun, r, 0, 12.*cos(3.*theta)), theta, 0, pi./6);
A=2.*q;
fprintf ('The shaded area is %.2f \n', A)
```
The shaded area is 37.70.

3.3 The area of the Snail shell

The area of the Snail shell enclosed by the positive x-axis and spiral $r = \dfrac{4\theta}{3}$, $0 \le \theta \le 2\pi$ can be calculated.

The graph of the area of the Snail shell enclosed by the positive x-axis and spiral can be expressed by using MATLAB code.

```
clc, clear, close all;
```

```
t=0:2*pi/100:2*pi;
r=4*t/3;
figure;
polar (t, r, 'm');
```
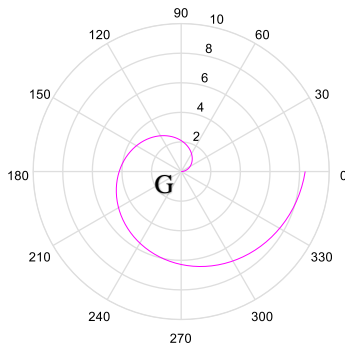

Fig 6 The Area of the Snail Shell.

The area is $\displaystyle\int_0^{2\pi}\int_0^{\frac{4\theta}{3}} r\,dr\,d\theta$ $= \displaystyle\int_0^{2\pi}\left[\frac{r^2}{2}\right]_{r=0}^{r=\frac{4\theta}{3}} d\theta$

$= \dfrac{1}{2}\displaystyle\int_0^{2\pi}\dfrac{16\theta^2}{9}d\theta$ $= \dfrac{8}{9}\displaystyle\int_0^{2\pi}\theta^2\,d\theta = \dfrac{64\pi^3}{27} \approx 73.50$

This can be calculated by MATLAB code.

```
clc, clear;
syms theta r
polarfun= @ (theta, r) r;
A=int (int (polarfun, r, 0, (4.*theta)./3), theta, 0, 2.*pi);
fprintf ('The Area is %.2f \n', A)
The Area is 73.50.
```

4. VOLUMES OF THE BOUNDED REGIONS

If $f(x,y)$ is positive and continuous over R, then the volume of the solid region between R and the surface $z = f(x,y)$ to be $\displaystyle\iint_R f(x,y)dA.$ If R is a region bounded 'above' and 'below' by the curves $y = g_2(x)$ and $y = g_1(x)$, and on the sides by the lines $x = a$, $x = b$ we may again calculate the volume by the method of slicing [Thomas (1914)]. We calculate the cross-sectional area

$$A(x) = \int_{y=g_1(x)}^{y=g_2(x)} f(x,y)dy \qquad (4)$$

and then integrate $A(x)$ from $x = a$ to $x = b$, to get the volume as an iterated integral:

$$V = \int_a^b A(x)dx = \int_a^b\int_{g_1(x)}^{g_2(x)} f(x,y)\ dy\,dx. \qquad (5)$$

Similarly, if R is a region bounded by the curves $x = h_2(y)$ and $x = h_1(y)$, and the lines $y = c$ and $y = d$, then the volume calculated by slicing is given by the iterated integral

$$V = \int_c^d\int_{h_1(y)}^{h_2(y)} f(x,y)\ dx\,dy. \qquad (6)$$

4.1 The volume of the wedgelike solid

We can find the volume of the wedgelike solid that lies beneath the surface $z = 16 - x^2 - y^2$ and above the region R bounded by the curve $y = 2\sqrt{x}$ the line $y = 4x - 2$ and the x-axis.

The graph of the volume of the wedgelike solid can be illustrated by using MATLAB code.

```
clc, clear, close all;
x=0:0.01:1;
y=0:0.1:2;
[X, Y] = meshgrid (x, y);
Z=16-X.^2-Y.^2;
figure, meshz (X, Y, Z);
xlabel ('x');
ylabel ('y');
zlabel ('z');
title ('Volume of Wedgelike solid');
```
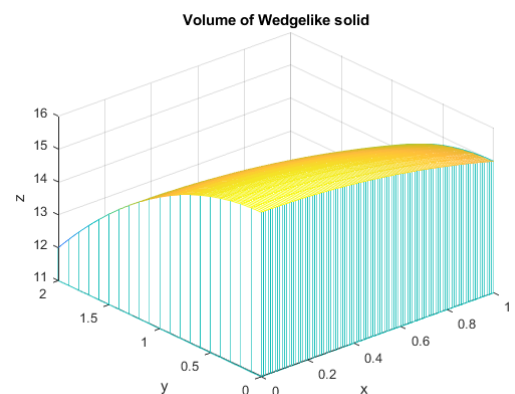

Fig 7 The Region of the Wedgelike Solid.

We integrate in the order $dx\,dy$ the integration will be required because x varies from $x = \dfrac{y^2}{2}$ to $x = \dfrac{(y+2)}{4}$ for $0 \le y \le 2.$ The volume is calculated as the iterated integral:

The volume is $\displaystyle\iint_R \left(16 - x^2 - y^2\right)dA$

$$= \int_0^2\int_{\frac{y^2}{2}}^{\frac{y+2}{4}} \left(16 - x^2 - y^2\right)dx\,dy = \frac{20803}{1680} \approx 12.38.$$

This can be calculated by MATLAB code.

```
clc, clear;
syms x y
fun=@ (x, y)16-x.^2-y.^2;
q=int (int (fun, x, y.^2. / 4,(y+2)./4),y,0,2);
fprintf ('The Volume is %.2f  \ n', q)
The Volume is 12.38.
```

4.2 The volume of noncircular right cylinder

The volume of noncircular right cylinder enclosed by the lemniscate $r^2 = \cos 2\theta$ is the base of a solid right cylinder whose top is bounded by the sphere $z = \sqrt{2 - r^3}$ can be found.

The graph of the volume of noncircular right cylinder enclosed by the lemniscate can be illustrated by using MATLAB code.

```
clc, clear, close all;
t=0:2*pi/100:2*pi;
r=sqrt(2*cos(2*t));
figure;
polar (t, r, 'b');
```
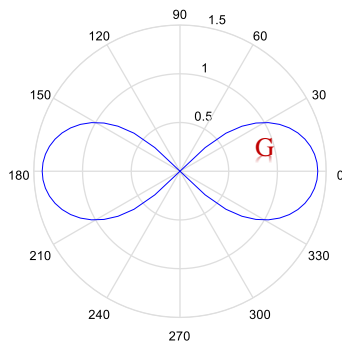

Fig 8 The Volume of Lemniscate.

The volume is $4\displaystyle\int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2\cos 2\theta}} r\sqrt{2-r^2}\, dr\, d\theta$

$$= 4\int_0^{\frac{\pi}{4}} \left[-\frac{1}{3}\left(2-r^2\right)^{\frac{3}{2}} \right]_0^{\sqrt{2\cos 2\theta}} d\theta = -\frac{4}{3}\int_0^{\frac{\pi}{4}} \left[\left(2-2\cos 2\theta\right)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right] d\theta$$

$$= -\frac{4}{3}\int_0^{\frac{\pi}{4}} (2)^{\frac{3}{2}} \left[\left(1-\cos 2\theta\right)^{\frac{3}{2}} - 1 \right] d\theta = \frac{40\sqrt{2} - 64 + 6\sqrt{2}\pi}{9} \approx 2.14.$$

This can be calculated by MATLAB code.

```
clc, clear;
syms theta r
polarfun =@ (theta, r) r.*(sqrt(2-r.^2));
q=int(int(polarfun,r,0,sqrt(2.*cos(2.*theta))),theta,0,pi./4);
V=4. *q;
fprintf ('The Volume is %.2f \n', V)
The Volume is 2.14.
```

5. EXPERIMENT APPARATUS

The area of the asphalt section on Union Road by double integral can be calculated. The road is defined by $f(x, y) = y\sin(x+y)$ within the rectangle $0 \le x \le 0.1$ and $0 \le y \le 5$, where x and y are measured in miles.

The graph of the area of the asphalt section on Union Road can be illustrated by using MATLAB code.

```
clc, clear, close all;
x=0:0.001:0.1;
y=0:0.01:5;
[X, Y] = meshgrid (x, y);
z=Y.*sin(X+Y);
figure, mesh (X, Y, z);
xlabel ('x');
ylabel ('y');
zlabel ('z');
```
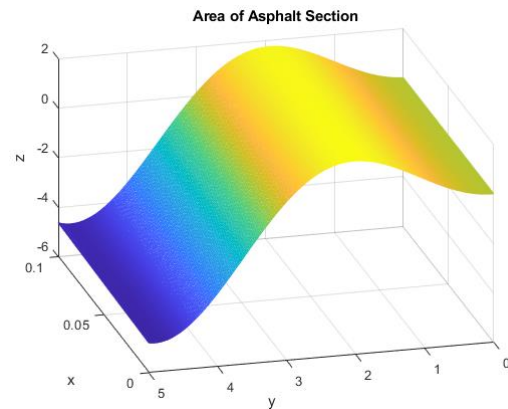

Fig 9 The Area of the asphalt section on Union Road.

The Area is $\displaystyle\iint_R y\sin(x+y)\, dA = \int_0^{0.1} \int_0^5 y\sin(x+y)\, dy\, dx$

$$= \int_0^{0.1} \left[-y\cos(x+y) + \sin(x+y) \right]_0^5 dx$$

$$= \int_0^{0.1} \left(-5\cos(x+5) + \sin(x+5) - \sin x \right) dx$$

$$= \left[-5\sin(x+5) - \cos(x+5) + \cos x \right]_0^{0.1}$$

$$\approx -0.2649.$$

This can be calculated by MATLAB code.

```
clc, clear;
syms x y
fun=@ (x, y) y.*sin(x+y);
A=int (int (fun, y,0,5), x,0,0.1);
fprintf ('The Area is %.4f\n', A)
The Area is -0.2649.
```

6. CONCLUSIONS

We have presented solving the problems of bounded regions in real life by using double integrals. We use the double integral methods to find the areas of the bounded regions between two curves, between the line and the curves and the volume of the wedgelike solid and lemniscate. We have solved the population density of a certain bacterium on the xy-plane and the planar region on Earth. By using the double integral, we have computed the areas and volumes of the objects in real life. We have discussed the comparison of solving the areas and the volumes with formulas and MATLAB

code in each example. We use these ideas later to find the design for engineering.

ACKNOWLEDGEMENT

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Solving *RC, LC, RLC*-Circuits Problems by Using Laplace Transforms

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ABSTRACT: The Laplace transforms are important basis in the engineer fields. In this paper we express about the Laplace transform and some of their applications in electric circuits. We use the first and second shifting theorems with the Linearity of the Laplace transform. We also discuss the Laplace transform of derivatives and integral. Moreover, we solve the problems in response of *RC, LC* and *RLC*-circuits to a single rectangular wave by using the Laplace transform.

KEYWORDS: *linearity, s-shifting, time shifting, Heaviside function, RLC-circuit.*

1. INTRODUCTION

Laplace transforms are invaluable for any engineer's mathematical toolbox as they make solving area of engineering and physics. The Laplace transforms is an integral transform because it changes a function in one space to a function in another space by a process of integration. In section 2, we will discuss the Laplace transform with linearity and the first shifting theorem. [Kreyszig (2011)]. We express the Unit step function (Heaviside function) to solve the *RC, LC, RLC*- electric circuits with the second shifting theorem in section 3.

2. LAPLACE TRANSFORM AND ITS INVERSE

If $f(t)$ is a function defined for all $t \geq 0$, its Laplace transform is the integral of $f(t)$ times e^{-st} from $t = 0$ to ∞. It is a function of s, say, $F(s)$, and is denoted by $\mathcal{L}(f)$; thus

$$F(s) = \mathcal{L}(f) = \int_0^\infty e^{-st} f(t) dt. \qquad (1)$$

Here we must assume that $f(t)$ is such that the integral exists.

Furthermore, the given function $f(t)$ in (1) is called the inverse transform of $F(s)$ and is denoted by $\mathcal{L}^{-1}(F)$ and we write

$$f(t) = \mathcal{L}^{-1}(F). \qquad (2)$$

From (1) and (2), we have

$$\mathcal{L}^{-1}(\mathcal{L}(f)) = f \text{ and } \mathcal{L}(\mathcal{L}^{-1}(F)) = F. \qquad (3)$$

2.1 Linearity of the Laplace transform

The Laplace transform is a linear operation; that is, for any function $f(t)$ and $g(t)$ whose transforms exist and any constants a and b the transform of $af(t) + bg(t)$ exists, and

$$\mathcal{L}\{a(f(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\}. \qquad (4)$$

2.2 First shifting theorem, s-shifting

If $f(t)$ has the transform $F(s)$, where $s > k$ for some k, then $e^{at}f(t)$ has the transform $F(s - a)$, where $(s - a) > k$. In formulas,

$$\mathcal{L}\{e^{at}f(t)\} = F(s - a), \qquad (5)$$

if we take the inverse on both sides,

$$e^{at}f(t) = \mathcal{L}^{-1}\{F(s - a)\}. \qquad (6)$$

2.3 Laplace transform of derivatives

The transforms of the first and second derivatives of $f(t)$ satisfy

$$\mathcal{L}(f') = s\mathcal{L}(f) - f(0). \qquad (7)$$
$$\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0). \qquad (8)$$

2.4 Laplace transform of integral

Let $F(s)$ denote the transform of a fuction $f(t)$ which is piecewise continuous for $t \geq 0$,

$\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}F(s)$, thus

$$\int_0^t f(\tau)d\tau = \mathcal{L}^{-1}\left\{\frac{1}{s}F(s)\right\}. \qquad (9)$$

2.5 Unit step function (Heaviside function)

The unit step function or Heaviside function $u(t - a)$ is 0 for $t < a$, has a jump of size 1 at $t = a$, in a formula;

$$u(t - a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t > a \end{cases} \qquad a \geq 0. \qquad (10)$$
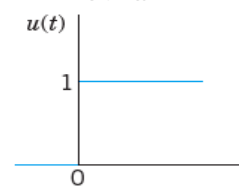
Fig. 1. Unit Step function $u(t)$
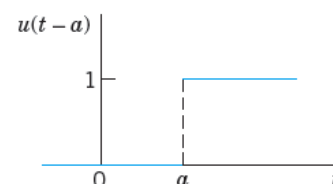
Fig. 2. Unit step function $u(t - a)$

Figure 1 shows the special case $u(t)$, which has its jump at zero, and Figure 2, the general case $u(t - a)$ for an arbitrary positive a. The transform of $u(t - a)$ follows

$\mathcal{L}\{u(t - a)\} = \int_0^\infty e^{-st} u(t - a)dt = \int_a^\infty e^{-st} \cdot 1 dt$,

here the integration begins at $t = a (\geq 0)$ because $u(t - a)$ is 0 for $t < a$. Hence

$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$ 　　　　　$(s > 0).$ 　　　(11)

2.6 Second shifting theorem; time shifting

If $f(t)$ has the transform $F(s)$, then the "shifted function"

$$\tilde{f}(t) = f(t-a)u(t-a)$$
$$= \begin{cases} 0 & \text{if } t < a \\ f(t-a) & \text{if } t > a \end{cases} \quad (12)$$

has the transform $e^{-as}F(s)$.

That is if $\mathcal{L}\{f(t)\} = F(s)$, then

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s). \quad (13)$$

Or, if we take the inverse on both sides, we can write

$$f(t-a)u(t-a) = \mathcal{L}^{-1}\{e^{-as}F(s)\}. \quad (14)$$

If the conversion of $f(t)$ to $f(t-a)$ is inconvenient, replace it by

$$\mathcal{L}\{f(t)u(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}. \quad (15)$$

3. DESIGNING A MODEL OF ELECTRIC CIRCUITS

Figure 3 shows an *RLC*-circuit, as it occurs as a basic building block of large electric networks in computers and elsewhere. An *RLC*-circuit is obtained from an *RL*-circuit by adding a capacitor. We obtain the model of the *RLC*-circuit simply by adding the voltage drop $\frac{Q}{C}$ across the capacitor. C (F, farads) is the capacitance of the capacitor. Q (coulombs) is the charge on the capacitor, related to the current by $I(t) = \frac{dQ}{dt}$ equivalently 　　　$Q(t) = \int I(t)dt.$

Assuming a sinusoidal EMF as in Figure 3, we have the model of the *RLC*-circuit.

$$E(t) = E_0 \sin \omega t$$
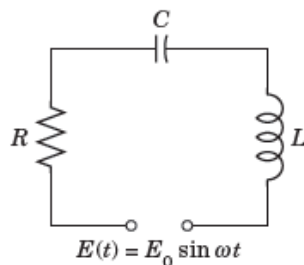
Fig.3. *RLC*-circuit

Table 1. Elements in an *RLC*-circuit

Name	Symbol	Notation	Unit	Voltage Drop
Ohm's Resistor	—\/\/\/—	R = Ohm's Resistor	ohms(Ω)	RL
Inductor	—oooo—	L = Inductance	henrys(H)	$L\frac{dI}{dt}$
Capacitor	—|(—	C = Capacitance	Farads(F)	$\frac{Q}{C}$

3.1 Response of an *RC*-circuit to a single rectangular wave

By using the Laplace transform, the current $i(t)$ in the circuit in Figure 4 with $R = 10\Omega$ and $C = 10^{-2}$F, where the current at $t = 0$ is assumed to be zero, and $v = 0$ if $t < 4$ and $14.10^6 e^{-3t}$V if $t > 4$ can be found.
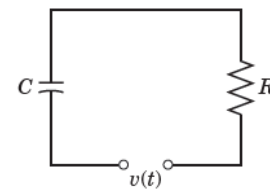
Fig. 4. *RC*-circuit

The equation for the current in the *RC*-circuit is

$$Ri(t) + \frac{1}{c}\int_0^t i(\tau)d\tau = v = \begin{cases} 0 & t < 4 \\ 14.10^6 e^{-3t}V, & t > 4 \end{cases}.$$

Using the Heaviside step function, we have

$v = 16.10^6 e^{-3t}u(t-4)$V.

It is given that $R = 10\Omega$, $C = 10^{-2}F$, so the above equation becomes

$$10i(t) + 10^2 \int_0^t i(\tau)d\tau = 14.10^6 e^{-3t}u(t-4).$$

Dividing by 10 and rewriting the right-hand side of the equation, we get

$$i(t) + 10^2 \int_0^t i(\tau)d\tau = 14.10^6 e^{-12}e^{-3(t-4)}u(t-4).$$

Taking the Laplace transform of the equation, using the Laplace transform of integral, the second shifting theorem and the linearity of the Laplace Transform (where $I(s) = \mathcal{L}\{i(t)\}$), we get

$$I(s) + 10\frac{I(s)}{s} = 14.10^5 \frac{e^{-12}e^{-4s}}{s+3}$$
$$\frac{s+10}{s}I(s) = 14.10^5 \frac{e^{-12}e^{-4s}}{s+3}$$
$$I(s) = 14.10^5 \frac{se^{-12}e^{-4s}}{(s+10)(s+3)}.$$

Taking the inverse Laplace transform and using its linearity and the second shifting theorem, we get

$$i(t) = \mathcal{L}^{-1}\left\{\left(\frac{10}{s+10} - \frac{3}{s+3}\right)2.10^5 e^{-12}e^{-4s}\right\}$$
$$= \left(10e^{-10(t-4)} - 3e^{-3(t-4)}\right)2.10^5 u(t-4)$$
$$= (10e^{-10t+40-12} - 3e^{-3t+12-12})2.10^5 u(t-4)$$
$$= (10e^{-10t+28} - 3e^{-3t})2.10^5 u(t-4)$$
$$= (10e^{-7t+28} - 3)2.10^5 e^{-3t}u(t-4)$$
$$= \left(10e^{-7(t-4)} - 3\right)2.10^5 e^{-3t}u(t-4).$$

3.2 Response of a *LC*-circuit to a single rectangular wave

By using the Laplace transform, the current $i(t)$ in the circuit in Figure 5, assuming zero initial current and charge on the capacitor and $L = 0.5$ H, $C = 0.05$ F, $v = 78\sin t$ V if $0 < t < \pi$ and 0 if $t > \pi$ can be calculated.
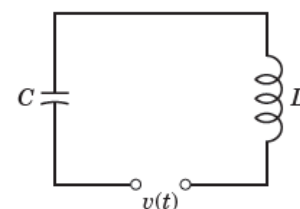
Fig. 5. *LC*-circuit

The equation for the current in the *LC*-circuit is

$$Li'(t) + \frac{1}{c}\int_o^t i(\tau)d\tau = v = \begin{cases} 78\sin t \text{ V}, & 0 < t < \pi \\ 0, & t > \pi \end{cases}.$$

Taking the first derivative of both sides, we get

$Li''(t) + \frac{1}{c}i(t) = v' = \begin{cases} 78\cos t \text{ V}, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$.

Using the Heaviside step function, we have

$v' = 78\cos t\,(1 - u(t - \pi))$

$= 78\cos t - 78\cos t\,u(t - \pi)$

$= 78\cos t - 78\cos\big((t - \pi) + \pi\big)u(t - \pi)$

$= 78\cos t + 78\cos(t - \pi)\,u(t - \pi)$

It is given that $L = 0.5$ H, $C = 0.05\text{F} = 5.10^{-2}\text{F}$ and that the initial current and charge is 0, so the above equation

$0.5\,i''(t) + 20i = 78\cos t + 78\cos(t - \pi)u(t - u)$,

$\qquad i(0) = i'(0) = 0$

$i''(t) + 40i = 156\cos t + 156\cos(t - \pi)u(t - \pi)$,

$\qquad i(0) = i'(0) = 0$.

Taking the Laplace transform of the equation. Using the Laplace transform of derivatives, the second shifting theorem and the linearity of the Laplace transform (where $I(s) = \mathcal{L}\{i(t)\}$), we get

$s^2 I(s) + 40I(s) = 156\frac{s}{s^2+1} + 156\frac{s}{s^2+1}e^{-\pi s}$

$I(s) = 156\frac{s}{(s^2+1)(s^2+40)} + 156\frac{s}{(s^2+1)(s^2+40)}e^{-\pi s}$.

Taking the partial fraction expansion of $I(s)$,

$I(s) = 4\frac{s}{s^2+1} - 4\frac{s}{s^2+40} + \left(4\frac{s}{s^2+1} - 4\frac{s}{s^2+40}\right)e^{-\pi s}$

Taking the inverse Laplace transform and using its linearity and the second shifting theorem, we get

$i(t) = \mathcal{L}^{-1}\left\{4\frac{s}{s^2+1} - 4\frac{s}{s^2+40} + \left(4\frac{s}{s^2+1} - 4\frac{s}{s^2+40}\right)e^{-\pi s}\right\}$

$\quad = 4\cos t - 4\cos\sqrt{40}\,t$

$\qquad + \left(4\cos t - 4\cos\sqrt{40}(t - \pi)\right)u(t - \pi)$

$\quad = 4\cos t - 4\cos\sqrt{40}\,t$

$\qquad -4\left(\cos t + \cos\sqrt{40}\,t(t - \pi)\right)u(t - \pi)$.

The equation can also be written as

$i(t) = \begin{cases} 4\cos t - 4\cos\sqrt{40}\,t, & 0 < t < \pi \\ -4\cos\sqrt{40}\,t - 4\cos\sqrt{40}(t - \pi), & t > \pi \end{cases}$.

3.3 Response of an *RLC*-circuit to a sinusoidal input acting over a time interval

By using the Laplace transform, the current $i(t)$ in the circuit in Figure 6, assuming zero initial current and charge and $R = 4\,\Omega, L = 1\text{H}, C = 0.05$ F, $v = 34e^{-t}$ V if $0 < t < 4$ and 0 if $t > 4$ can be evaluated.
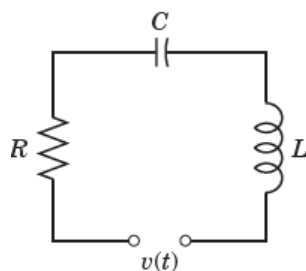
Fig. 6. *RLC*- circuit

The equation for the current in the *RLC*-circuit is

$Li'(t) + Ri(t) + \frac{1}{c}\int_0^t i(\tau)d\tau = \begin{cases} 34e^{-t}\text{V}, & 0 < t < 4 \\ 0, & t > 4 \end{cases}$.

Using the Heaviside step function,

$v = 34e^{-t}\left(1 - u(t - 4)\right)$

$\quad = 34e^{-t} - 34e^{-t+4-4}u(t - 4)$

$\quad = 34e^{-t} - 34e^{-4}e^{-(t-4)}u(t - 4)$.

It is given that $R = 4\,\Omega, L = 1$ H, $C = 0.25$ F and that the initial current and charge is 0, so the above equation becomes

$i'(t) + 4i(t) + 20\int_0^t i(\tau)d\tau$

$= 34e^{-t} - 34e^{-4}e^{-(t-4)}u(t - 4), i(0) = i'(0) = 0$.

Taking the Laplace transform of the equation, using the Laplace transform of derivatives, the Laplace transform of integral, the second shifting theorem and the linearity of the Laplace transform (where $I(s) = \mathcal{L}\{i(t)\}$),

$sI(s) + 4I(s) + 20\frac{I(s)}{s} = \frac{34}{s+1} - \frac{34e^{-4}e^{-4s}}{s+1}$

$(s^2 + 4s + 20)\frac{I(s)}{s} = \frac{34}{s+1} - \frac{34e^{-4}e^{-4s}}{s+1}$

$I(s) = \frac{34s}{(s+1)(s^2+4s+20)} - \frac{34e^{-4}se^{-4s}}{(s+1)(s^2+4s+20)}$.

Taking the partial fraction expansion of $I(s)$ and $s^2 + 4s + 20 = (s + 2)^2 + 4^2$, we get

$I(s) = \frac{2s+40}{s^2+4s+20} - \frac{2}{s+1} - \left(\frac{2s+40}{s^2+4s+20} - \frac{2}{s+1}\right)e^{-4}e^{-4s}$

$\quad = \frac{2(s+2)}{(s+2)^2+4^2} + \frac{36}{(s+2)^2+4^2} - \frac{2}{s+1}$

$\qquad + \left(\frac{2}{s+1} - \frac{2(s+2)}{(s+2)^2+4^2} - \frac{36}{(s+2)^2+4^2}\right)e^{-4}e^{-4s}$.

Taking the inverse Laplace transform and using its linearity and the second shifting theorem, we get

$i(t) = \mathcal{L}^{-1}\{\frac{2(s+2)}{(s+2)^2+4^2} + \frac{36}{(s+2)^2+4^2} - \frac{2}{s+1}$

$\qquad + \left(\frac{2}{s+1} - \frac{2(s+2)}{(s+2)^2+4^2} - \frac{36}{(s+2)^2+4^2}\right)e^{-4}e^{-4s}\}$

$\quad = 2\cos 4t\,e^{-2t} + 9\sin 4t\,e^{-2t} - 2e^{-t}$

$\qquad + (2e^{-t-1} - 2\cos 4(t - 4)\,e^{-2(t-4)}$

$\qquad - 9\sin 4(t - 4)\,e^{-2(t-4)})\,e^{-4}u(t - 4)$

$\quad = (2\cos 4t + 9\sin 4t)e^{-2t} - 2e^{-t}$

$\qquad + (2e^{-t} - 2\cos 4(t - 4)$

$\qquad + 9\sin 4(t - 4)\,e^{-2(t-2)})u(t - 4)$.

We can write the solution as

$i(t) = \begin{cases} (2\cos 4t + 9\sin 4t)e^{-2t} - 2e^{-2t}, & 0 < t < 4 \\ (2\cos 4t + 9\sin 4t)e^{-2t} \\ \quad -2\big(\cos 4(t - 4) + 9\sin 4(t - 4)\big)e^{-2(t-2)}, & t > 4 \end{cases}$.

Next, we can find the response of the *RLC*- circuit with

$R = 2\Omega, L = 1$ H, $C = 0.5\text{F}, v(t) = 1$ kV if $0 < t < 2$ and 0 if $t > 2$.

The model for the current $i(t)$ passing in *RLC*-circuit is

$$Li'(t) + Ri(t) + \frac{1}{c}\int_0^t i(\tau)d\tau = v(t).$$

Substituting $R = 2, L = 1, c = 0.5$ and

$v(t) = \begin{cases} 1000, & 0 < t < 2 \\ 0, & t > 2 \end{cases}$ in *RLC*-circuit, we get

$i' + 2i + \frac{1}{0.5}\int_0^t i(\tau)d\tau = \begin{cases} 1000, & 0 < t < 2 \\ 0, & t > 2 \end{cases}$.

So, we have

$v(t) = 1000\,(1 - u(t - 2)) + 0.\,u(t - 2)$

$\qquad = 1000(1 - u(t - 2))$.

Thus, the differential equation becomes

$i' + 2i + 2\int_0^t i(\tau)d\tau = 1000(1 - u(t - 2))$.

As the initial charge and current are zero, the initial conditions become $i(0) = 0$ and $i'(0) = 0$.

Applying the Laplace transform on both sides,

$sI(s) - I(0) + 2I(s) + 2(\frac{I(s)}{s}) = 1000\left(\frac{1}{s} - \frac{e^{-2s}}{s}\right)$.

Using the initial conditions, $i(0) = 0$ and $i'(0) = 0$,

$$I(s)\left(s + 2 + \frac{2}{s}\right) = \frac{1000}{s}(1 - e^{-2s})$$

$$I(s) = \frac{1000(1-e^{-2s})}{s^2+2s+2} = \frac{1000(1-e^{-2s})}{(s+1)^2+1}$$

$$I(s) = 1000\left(\frac{1}{(s+1)^2+1} - \frac{e^{-2s}}{(s+1)^2+1}\right).$$

Applying the inverse Laplace transform on both sides,

$$i(t) = 1000\mathcal{L}^{-1}\left(\frac{1}{(s+1)^2+1} - \frac{e^{-2s}}{(s+1)^2+1}\right).$$

Using the second shifting theorem,

$$i(t) = 1000e^{-t}\sin t - 1000u(t-2)e^{-(t-2)}\sin(t-2).$$

4. CONCLUSIONS

We have described the Laplace transform to find the current in the electric circuits. The modeling electric circuits will be profitable for electrical engineers and computer scientists. The Laplace transform can also be used extensively in electrical engineering. We conclude that this paper leads to the students to have the knowledge of the Laplace transforms and its advantages with the electric circuits.

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Solving the Assignment Problems in Social Life

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ABSTRACT: The aim of this paper is solving the assignment problems which is the most of the social life problems with minimum total cost and maximum profit by Hungarin method. The assignment technique which is applicable to a class of very practical problem to obtain the optimal assignment of tasks to resources. These problems find numerous applications in real life situations. The method of assignment problem is to assign a number of jobs to the equal number of facilities (machines or person or destinations) at a minimum cost or maximum profit in various diverse business situations. The interesting social life problems with balanced and unbalanced problems are solved.

KEYWORDS: *assignment problem, maximum profit, minimum cost, modified matrix, optimal assignment.*

1. INTRODUCTION

An assignment problem is a special class of linear programming problem where the objective is to minimize the cost or time of completing a number of jobs by a number of destinations. If n jobs have to be assigned to n workers or machines with unit cost or unit time of performing the job, we can use assignment model to get minimum cost. Similarly we can solve the maximum profit for each jobs on each facilities. In section 2, we will state the mathematical formulation of the assignment problem. In an assignment problem, the number of sources and the number of destinations are equal [Wagner (2012)]. Thus the cost matrix must be a square matrix. The distribution and the requirement value is exactly one. Hungarin method for solving the assignment problems is explained in section 3. Each assignment problem has associated with it a matrix. In this paper, we will express the two types of assignment problems namely; balanced and unbalanced assignment problems [Kalavathy (2000)].

2. MATHEMATICAL FORMULATION OF THE ASSIGNMENT PROBLEM

In this section, we will describe the model of assignment problem and state the mathematical formulation. It involves assignment of people to projects, jobs to machines, depots to destinations and teachers to classes etc., while minimizing the total assignment costs. Each assignment problem has a $n \times n$ cost matrix $[c_{ij}]$ of real numbers, where c_{ij} is the cost if the ith person is assigned to the jth job. The problem is to find an assignment so that the total cost of performing all jobs is minimum.

Mathematically, the assignment problem can be expressed as

Minimize $\quad z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \, x_{ij} \quad , i = 1, 2, ..., n; j = 1, 2, ..., n.$

Subject to the restrictions

$$x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person is assigned } j^{th} \text{ job} \\ 0 & \text{if not} \end{cases}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{and} \quad \sum_{i=1}^{n} x_{ij} = 1$$

where x_{ij} denotes that the jth job is to be assigned to the ith person.

3. HUNGARIN METHOD FOR SOLVING ASSIGNMENT PROBLEM

Step 1. Construct a cost matrix. If the cost matrix is not a square matrix then add a dummy row (column) with zero element.

Step 2. Subtract the minimum element in each row from all the elements of the respective rows of the given matrix.

Step 3. To get the modified matrix by subtracting the minimum element of each column from all the elements of the respective columns.

Step 4. Draw the minimum number of horizontal and vertical lines to cover all zeros in the resulting matrix. The minimum number of lines denoted by N and the order of matrix by n.

Case *i*. If $N = n$, then an optimal assignment can be obtained.

Case *ii*. If $N < n$, then go to step 5.

Step 5. To obtain the second modified matrix, subtract the smallest uncovered element from all uncovered elements and add the same element at the intersection of horizontal and vertical lines.

Step 6. Repeat the step 3 and 4 until we get the case *i* of step 4.

Step 7. Determine the rows successively until a row-wise exactly single zero is found. Encircle this row (◯) and cross off (✗) all other zeros in its column. Continue in this manner until all the rows have been determined. Repeat the same procedure for column also.

Step 8. Repeat step 6 successively until one of the following situation appears

(*i*) if no unmarked zero is left, then the process stop or

(*ii*) if there lies more than one of the unmarked zero in any column or row then, circle one of the unmarked zeros arbitrarily and mark a cross in the entries of remaining zeros in its row or column. Repeat the process until no unmarked zero is left in the resulting matrix.

Step 9. Thus exactly one marked circled zero in each row and column of the matrix is obtained. This assignment corresponding to these marked circled will give the optimal assignment.

We have to solve the objective is to maximize the profit. To solve this we first convert the given profit matrix into the cost matrix by subtracting all the elements from the highest element of the given matrix [Kalavathy (2000)].

4. PRATICAL ASSIGNMENT PROBLEMS

4.1 Example

Four taxis 1,2,3 and 4 in a travel agency are available and there are four foreigners *A*, *B*, *C* and *D*, requiring taxis. The distance between the taxis and the foreigners are shown in the matrix below, in kilometers. The agency wishes to assign the taxis to the foreigners so that the total distance travelled is a minimum.

$$\begin{array}{c} \textit{Foreigners} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{Taxis} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \left[\begin{array}{cccc} 10 & 25 & 15 & 20 \\ 15 & 5 & 30 & 15 \\ 35 & 12 & 20 & 24 \\ 17 & 25 & 24 & 20 \end{array} \right] \end{array}$$

To get the first modified matrix by substracting the minimum element from all elements in the respective rows and same with respective columns.

$$\begin{array}{c} \textit{Foreigners} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{Taxis} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \left[\begin{array}{cccc} 0 & 15 & 0 & 7 \\ 10 & 0 & 20 & 7 \\ 23 & 0 & 3 & 9 \\ 0 & 8 & 2 & 0 \end{array} \right] \end{array}$$

In this modified matrix, we draw the minimum number of the lines to cover all zero (horizontal or vertical).

$$\begin{array}{c} \textit{Foreigners} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{Taxis} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \left[\begin{array}{cccc} 0 & 15 & 0 & 7 \\ 10 & 0 & 20 & 7 \\ 23 & 0 & 3 & 9 \\ 0 & 8 & 2 & 0 \end{array} \right] \end{array}$$

Number of lines drawn to cover all zeros is 3=*N*. The order matrix is *n* = 4. Hence *N* < *n*.

To get the second modified matrix by substracting the smallest uncovered element from the remaining uncovered elements and add to the elements at the point of intersection of lines.

$$\begin{array}{c} \textit{Foreigners} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{Taxis} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \left[\begin{array}{cccc} 0 & 18 & 0 & 7 \\ 7 & 0 & 17 & 4 \\ 20 & 0 & 0 & 6 \\ 0 & 11 & 2 & 0 \end{array} \right] \end{array}$$

Number of lines drawn to cover all zeros is 4 = *N*. Thus *n* = *N*. Now we determined the optimum assignment.

$$\begin{array}{c} \textit{Foreigners} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \textit{Taxis} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \left[\begin{array}{cccc} ⓪ & 18 & ✗ & 7 \\ 7 & ⓪ & 17 & 4 \\ 20 & ✗ & ⓪ & 6 \\ ✗ & 11 & 2 & ⓪ \end{array} \right] \end{array}$$

Optimal assignment and optimum distance of assignment is

$$A \to 1, \ B \to 2, \ C \to 3, \ D \to 4.$$

Minimum total distance is 10+5+20+20=55 kilometers.

Now we will solve the maximization in assignment problems.

4.2 Example

In a certain Technological University, the department of engineering mathematics has four teachers to be assigned to four different courses (Engineering Mathematics I, II, III, IV). All of the teachers have taught the courses in the last year and have been evaluated by the students. The rating for each teacher for each course is given in the following matrix, a perfect score is 100. The head of department wants to conclude the optimal assignment of teachers to courses that will maximize the overall average evaluation.

Courses

$$\begin{array}{c} & \begin{array}{cccc} I & II & III & IV \end{array} \\ \text{Teachers} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 90 & 75 & 80 & 81 \\ 80 & 85 & 75 & 78 \\ 85 & 82 & 70 & 82 \\ 83 & 81 & 82 & 85 \end{array}\right] \end{array}$$

To maximize the evaluation, we convert it into cost matrix which can be minimized. To convert it into cost matrix, we subtract all the elements from the element 90.

Courses

$$\begin{array}{c} & \begin{array}{cccc} I & II & III & IV \end{array} \\ \text{Teachers} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 15 & 10 & 9 \\ 10 & 5 & 15 & 12 \\ 5 & 8 & 20 & 8 \\ 7 & 9 & 8 & 5 \end{array}\right] \end{array}$$

To get the first modified matrix, substract the minimum element from all the elements in the respective rows and columns.

Courses

$$\begin{array}{c} & \begin{array}{cccc} I & II & III & IV \end{array} \\ \text{Teachers} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 15 & 7 & 9 \\ 5 & 0 & 7 & 7 \\ 0 & 3 & 12 & 3 \\ 2 & 4 & 0 & 0 \end{array}\right] \end{array}$$

We draw the minimum number of lines to cover all zeros.

Courses

$$\begin{array}{c} & \begin{array}{cccc} I & II & III & IV \end{array} \\ \text{Teachers} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 15 & 7 & 9 \\ 5 & 0 & 7 & 7 \\ 0 & 3 & 12 & 3 \\ 2 & 4 & 0 & 0 \end{array}\right] \end{array}$$

$N = 3 < n = 4$. To get the second modified matrix, we substract the smallest uncovered element from the remaining uncovered elements and add to the elements at the point of intersection of lines.

Courses

$$\begin{array}{c} & \begin{array}{cccc} I & II & III & IV \end{array} \\ \text{Teachers} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 12 & 4 & 6 \\ 8 & 0 & 7 & 7 \\ 0 & 0 & 9 & 0 \\ 5 & 4 & 0 & 0 \end{array}\right] \end{array}$$

$N = n = 4$. Hence we make the assignment.

Courses

$$\begin{array}{c} & \begin{array}{cccc} I & II & III & IV \end{array} \\ \text{Teachers} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} ⓪ & 12 & 4 & 6 \\ 8 & ⓪ & 7 & 7 \\ ✗ & ✗ & 9 & ⓪ \\ 5 & 4 & ⓪ & ✗ \end{array}\right] \end{array}$$

Therefore the optimal assignment is

$$1 \rightarrow I, 2 \rightarrow II, 3 \rightarrow IV, 4 \rightarrow III .$$

The maximum overall average evaluation is 90+85+82+82=339.

4.3 Example

A marketing manager has 5 salesmen and there are 5 sales districts. Considering the capabilities of the salesmen and the nature of districts, the estimates made by the marketing manager for the sales per month (in dollars) for each salesmen in each district would be as follows. The maximum profit can be calculated.

Districts

$$\begin{array}{c} & \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \text{Salesmen} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \left[\begin{array}{ccccc} 30 & 37 & 40 & 28 & 40 \\ 40 & 24 & 27 & 21 & 36 \\ 40 & 32 & 33 & 30 & 35 \\ 25 & 38 & 40 & 36 & 36 \\ 29 & 62 & 41 & 34 & 39 \end{array}\right] \end{array}$$

To maximize the profit, we convert it into cost matrix which can be minimized. To convert it into cost matrix, we subtract all the elements from the element 62.

Districts

$$\begin{array}{c} & \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \text{Salesmen} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \left[\begin{array}{ccccc} 32 & 25 & 22 & 34 & 22 \\ 22 & 38 & 35 & 41 & 26 \\ 22 & 30 & 29 & 32 & 27 \\ 37 & 24 & 22 & 26 & 26 \\ 33 & 0 & 21 & 28 & 23 \end{array}\right] \end{array}$$

To get the first modified matrix, substract the minimum element from all the elements in the respective rows and columns.

Districts

$$\begin{array}{c} & \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \text{Salesmen} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \left[\begin{array}{ccccc} 10 & 3 & 0 & 8 & 0 \\ 0 & 16 & 13 & 15 & 4 \\ 0 & 8 & 7 & 6 & 5 \\ 15 & 2 & 0 & 0 & 4 \\ 33 & 0 & 21 & 24 & 23 \end{array}\right] \end{array}$$

We draw the minimum number of lines to cover all zeros.

Districts

$$\begin{array}{c} & I \quad II \quad III \quad IV \quad V \\ \begin{array}{c} A \\ B \\ \text{Salesmen } C \\ D \\ E \end{array} & \left[\begin{array}{ccccc} 10 & 3 & 0 & 8 & 0 \\ 0 & 16 & 13 & 15 & 4 \\ 0 & 8 & 7 & 6 & 5 \\ 15 & 2 & 0 & 0 & 4 \\ 33 & 0 & 21 & 24 & 23 \end{array}\right] \end{array}$$

$N = 4 < n = 5$. To get the second modified matrix, we substract the smallest uncovered element from the remaining uncovered elements and add to the elements at the point of intersection of lines.

Districts

$$\begin{array}{c} & I \quad II \quad III \quad IV \quad V \\ \begin{array}{c} A \\ B \\ \text{Salesmen } C \\ D \\ E \end{array} & \left[\begin{array}{ccccc} 14 & 3 & 0 & 8 & 0 \\ 0 & 12 & 9 & 11 & 0 \\ 0 & 4 & 3 & 2 & 1 \\ 19 & 2 & 0 & 0 & 4 \\ 37 & 0 & 21 & 24 & 23 \end{array}\right] \end{array}$$

$N = n = 5$. Hence we make the assignment.

Districts

$$\begin{array}{c} & I \quad II \quad III \quad IV \quad V \\ \begin{array}{c} A \\ B \\ \text{Salesmen } C \\ D \\ E \end{array} & \left[\begin{array}{ccccc} 14 & 3 & ⓪ & 8 & ⊠ \\ ⊠ & 12 & 9 & 11 & ⓪ \\ ⓪ & 4 & 3 & 2 & 1 \\ 19 & 2 & ⊠ & ⓪ & 4 \\ 37 & ⓪ & 21 & 24 & 23 \end{array}\right] \end{array}$$

The optimum assignment is

$$A \rightarrow III, B \rightarrow V, C \rightarrow I, D \rightarrow IV, E \rightarrow II.$$

Maximum profit is 40+36+40+36+62=214 $.

4.3 Unbalanced assignment problem

If the cost matrix is not square matrix, that is, the number of rows is not equal to the number of columns, then it is an unbalanced assignment problem. To make it balanced we add a dummy row or dummy column with all the entries as zero.

A company has 4 machines to do 3 jobs with the restriction that exactly one machine is allowed for each job. The assignment which will minimize the total cost can be solved. The associate costs (dollars) are as follows:

Machines

$$\begin{array}{c} & W \quad X \quad Y \quad Z \\ \begin{array}{c} 1 \\ \text{Jobs } 2 \\ 3 \end{array} & \left[\begin{array}{cccc} 60 & 50 & 70 & 80 \\ 80 & 30 & 90 & 50 \\ 50 & 60 & 40 & 70 \end{array}\right] \end{array}$$

Since the number of jobs is less than the number of machines, we add a dummy row (job) 4 with all the elements 0.

Machines

$$\begin{array}{c} & W \quad X \quad Y \quad Z \\ \begin{array}{c} 1 \\ 2 \\ \text{Jobs } 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 60 & 50 & 70 & 80 \\ 80 & 30 & 90 & 50 \\ 50 & 60 & 40 & 70 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

To get the first modified matrix, we will subtract the minimum element in each row from all the elements in its row.

Machines

$$\begin{array}{c} & W \quad X \quad Y \quad Z \\ \begin{array}{c} 1 \\ 2 \\ \text{Jobs } 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 10 & 0 & 20 & 30 \\ 50 & 0 & 60 & 20 \\ 10 & 20 & 0 & 30 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Since each column has minimum element, we draw the minimum number of lines to cover all zeros.

Machines

$$\begin{array}{c} & W \quad X \quad Y \quad Z \\ \begin{array}{c} 1 \\ 2 \\ \text{Jobs } 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 10 & 0 & 20 & 30 \\ 50 & 0 & 60 & 20 \\ 10 & 20 & 0 & 30 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$N = 3 < n = 4$. To get the second modified matrix by substracting the smallest uncovered element from the remaining uncovered elements and add to the elements at the point of intersection of lines.

Machines

$$\begin{array}{c} & W \quad X \quad Y \quad Z \\ \begin{array}{c} 1 \\ 2 \\ \text{Jobs } 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 0 & 20 & 20 \\ 40 & 0 & 60 & 10 \\ 0 & 20 & 0 & 20 \\ 0 & 10 & 10 & 0 \end{array}\right] \end{array}$$

Here the number of lines drawn to cover all zero is order of matrix. $N = n = 4$. Hence we make the assignment.

Machines

$$\begin{array}{c} & W \quad X \quad Y \quad Z \\ \begin{array}{c} 1 \\ 2 \\ \text{Jobs } 3 \\ 4 \end{array} & \left[\begin{array}{cccc} ⓪ & ⊠ & 20 & 20 \\ 40 & ⓪ & 60 & 10 \\ ⊠ & 20 & ⓪ & 20 \\ ⊠ & 10 & 10 & ⓪ \end{array}\right] \end{array}$$

Since 4 is dummy job, machine Z has assigned no job. The optimum assignment is

$$1 \rightarrow W, 2 \rightarrow X, 3 \rightarrow Y.$$

Therefore the optimum cost is 60+30+40=130 $.

5. CONCLUSIONS

I have discussed solving the assignment problems in social life with Hungarin method. The assignment problem is a combinatorial optimization problem that is flexible it can be used as an approach to model any social life problem. Sometimes total demand is frequently not equal to total supply. These unbalanced problems can be handled easily by adding dummy destinations. Assignment problem can be found applicability in various business situation. Some of its application areas are in assigning machines to factory orders, in assigning marketing people to sales territories, in assigning teachers to classes and many more other applications. The results showed that the solving assignment problems is very useful for social life.

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Mathematical Performance Analysis of Engineering Students using Descriptive Statistics and Absorbing Markov

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ABSTRACT*:* This research study analyses the profiles and trend of mathematical performance and retainage of engineering students. Primary compiled data were collected and analysed with descriptive and inferential statics using SPSS software. Upon analysis, it is found that the number of students decreases batches by batches and academic years by year. Approximately 80% of students completed engineering mathematics courses after their four years study. Students from Civil and Electrical Power engineering majors are best in mathematical performance. It is also found that mathematical performance of students first semesters of each years is better than those of second years. Without studying and understanding mathematics, it is not possible for engineering student to study and solve engineering problems. Therefore, this type of research is imperative to conduct for analysing the mathematical performance status and trends engineering students for other universities in order that the management of respective universities can improve relevant facilities and support students perusing quality education, leading to nurturing qualified students-engineers.

KEYWORDS*: profile, mathematical performance, SPSS, Quality, and Nurturing*

1. INTRODUCTION

This research paper intends to analyses the profile data and trend of mathematical study by engineering students using descriptive statistics analysis. Lot of researches have been conducted to explore the factors affecting mathematical performance of students. Analysis results and reports are useful in management decision making and exploring the factors which have significant impact on the performance of engineering students.

Several researches have been conducted to identify the key factors influencing on the education or specific field area performance of students. The study by Amanda M. Vandervelde investigates the utility of various curriculum-based assessment and measurement estimates of mathematics performance for predicting functional outcomes. As the professional fields are different and the factors influencing on performance may be different, there is a need to conduct research to explore key factors influencing on performance of students and their significance.

SPSS statistical software is most commonly used for performing statistical analysis of collected primary complied data.

1.1 Background of the study

Various engineering principles have been applied to invent and solve engineering problems. There is no engineering principle, theory and assumption without the application of mathematics. Therefore, engineering mathematics is the main subject that engineering students must be acquaint with and able to apply problems. In producing qualified engineers, understanding, applicability and mathematical performance is necessary and important not only during engineering study but also daily engineering profession. Good academic achievement is the requisite for further studies and marketability for future career. Thus, students must put the serious and greatest effort in their study for scoring good grades and to prepare themselves for future opportunities in their career. Students' academic performance is affected by several factors such as university learning infrastructure, parental motivation and support, students' learning skills, teachers' quality, teaching material quality, laboratory and research facilities etc [Oyediran (2018)].

Alos et al highlighted and emphasized the importance of qualified and experienced teachers in teaching and said that success of any program is subject to the ability and qualification of the teachers. Failures in having that will lead to the whole system failure. This research is aimed to identify the factors affecting on the academic performance of engineering students as case study. Exploring the factors would help in giving recommendations that would help promote a better academic performance in the study area and all the universities of Myanmar at large [Joshi (2015)].

2. LITERATURE REVIEW

The following statistical background theories are studied for the purpose of supporting the research study.

2.1 Students' academic performance

Gallaher and Darling utilized grade point average (GPA) to measure students' academic performance focusing on performance of students for the semester. In the same time, other researchers used examination

results of previous year while focusing on performance for the specific subject or year [Naqvi (2006)]**.**

2.2 Descriptive analysis

Descriptive analysis is conducted to know the distribution and pattern of collected data, in order to identify relation among variables, hence providing information for conducting further statistical analyses. There are two types of descriptive analysis which are descriptive analysis for each individual variable and that for combinations of variables. Variables are classified into quantitative variable (e.g. age, year and weight etc.) and categorical variables (e.g. colour, marital status, gender etc.). In detail statistical analysis, the relationship between independent variables and dependents variables are studied [Kaushik (2014)].

3. RESEARCH OBJECTIVES

The objectives of the research study are to analyse the profile data of engineering students and trend of study in engineering performance of engineering students.

4. RESEARCH QUESTIONS

The research questions for this study are "What are the profiles data of engineering students under this study" and "What are the major trends in engineering mathematics study".

5. RESEARCH METHODOLOGY

Quantitative research is mainly based on the calculation and interpretation of numbers and figures. The analysis results are used to quantify and interpret opinions, attitudes, behaviours, and other described variables with the goal to help or refute hypotheses about a specific phenomenon, and potentially contextualize the outcomes and results from learning about the sample in a wider population (or specific groups).

Under this research study, a total of (870) number of students from all three batches of eight engineering disciplines are included and analysed. To have widest coverage and using sample calculation with confidence interval of (95%) and marginal error of 5%, repeated datas of (517) numbers of student's samples from batches (1&3) are selected and collected data proportionately. The data are collected through academic records of four years and eight semesters where the students study engineering mathematics subject.

The collected compiled student data are grouped into their enrolment for particular semester as present (Pr), student who fail in the semester exam as fail (F), students who did excellent as (E) and students who transferred from and repeated in the same semesters as Withdrawn (W). Descriptive statistics analysis was conducted with the help of SPSS (Statistical Package for the Social Sciences) software [Joshi (2015)].

6. ANALYSIS, RESULTS AND DISCUSSION

As mentioned above, collected data were analysed using SPSS (version 23.0) statistical software. Analysis screen shots are presented below as proven evidence.

6.1 Profile analysis

All the collected compiled data are inputted in SPSS program in variables and data view. Descriptive statistics analysis is conducted based on the frequencies of the variables under analysis.

The following are SPSS output screen shots tables for Major and Results variables for reference. The results for all the variables are tabulated in the below tables and pictures.

The profile data of First year, first semester (Y1S1) students of batch one are analysed as below. Similar analysis shall be performed for all other years and semester to know how the distribution and trend of student populations.

Table 1.   Table 1. SPSS output for student by major (Y1S1)

	Frequency	Percent	Valid Percent	Cumulative Percent
Civil	40	15.6	15.6	15.6
Arch	29	11.3	11.3	26.8
Mech	41	16.0	16.0	42.8
EC	41	16.0	16.0	58.8
EP	35	13.6	13.6	72.4
IT	37	14.4	14.4	86.8
McE	20	7.8	7.8	94.6
Che	14	5.4	5.4	100.0
Total	257	100.0	100.0	
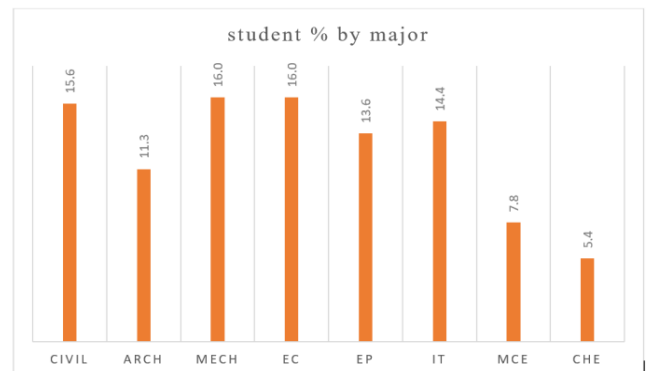
Fig 1. SPSS screen shot for descriptive analysis (Y1S1)

As in the analysis result of the above table and figure, it is obvious that the student population is the highest in Mechanical and Electronics Engineering disciplines and the lowest in the Chemical engineering. Population in Civil and Information Technology Engineering Disciplines stands the second.

Secondly, profile analysis is conducted on the population of total in and out (Y1S1 to Y4S2) as in the below table. Analysis is performed on the data on year-1 semester-1 student population and shall also be conducted.

Table 2.   Table 2. Student population by batch one

Student number	Batch One
Total In	257
Total Out	174
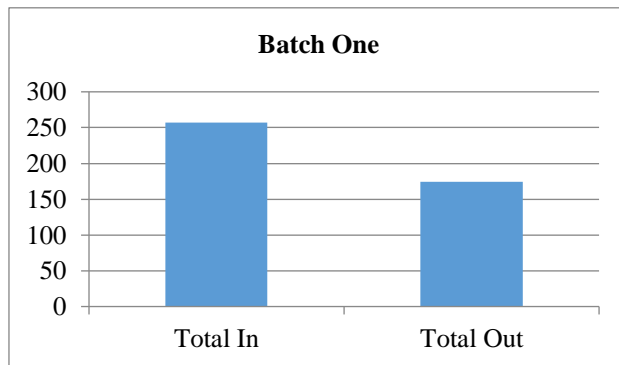
Fig 2. Student population by batch one

Table 3.   Table 3. Student by exam result (Y1S1)

	Frequency	Percent	Valid Percent	Cumulative Percent
Excellent	118	46	46	46
Pass (Only)	120	46.7	46.7	92.7
Fail	19	7.3	7.3	100
Total	257	100	100	
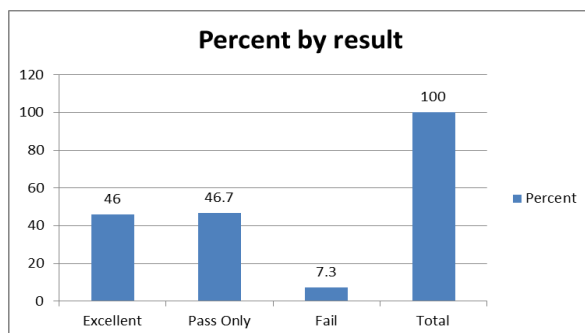
Fig 3. Student examination result (Y1S1)

Analysis on examination result data of first year, first semester is conducted as in the above table and figures. Based on the result, it can be seen that (46%) of students pass with excellent scoring at least 85 points, while (46.7 %) just normally pass and (7.3 %) fail in the exam.

6.2 Trend analysis

The following are figures to illustrate the analysis results of variables considered in the SPSS descriptive analysis.

In order to understand the trend of the number of students, frequencies analysis is conducted on the population of first year, first semester students who started studying engineering mathematics and fourth year, second semester students who completed four years engineering mathematics study during their undergraduate engineering program. For this research, analysis is conducted only on batch one of the three batches.

Table 4.   Table 4(a). Student population by major (Y1S1)

Major	Present number of students	Student %
Civil	40	16%
Arch	29	11%
MECH	41	16%
EC	41	16%
EP	35	14%
IT	37	14%
McE	20	8%
ChE	14	5%
Total	257	100%
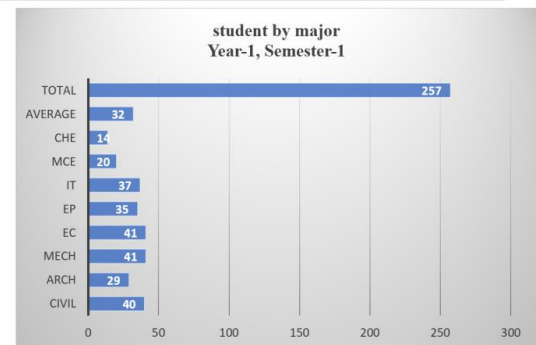
Fig 4(a). Student population by major (Y1S1)

The above table and figure show that there are (257) students enrolling at first year first semester with biggest student numbers in Mechanical and Electronics engineering majoring.

Table 5.   Table 4(b). Student population by major (Y4S2)

Major	Present number of students	Student %
Civil	33	19%
Arch	22	13%
MECH	23	13%
EC	23	13%
EP	27	16%
IT	29	17%
McE	11	6%
ChE	6	3%
	174	100%
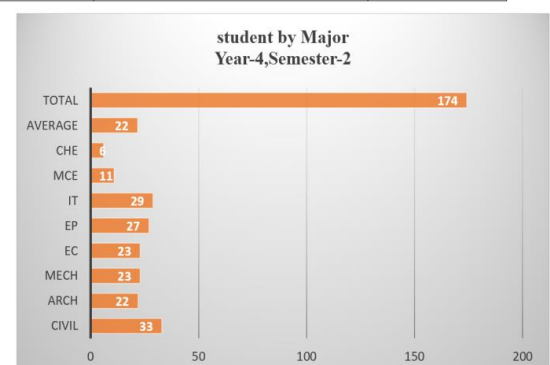
Fig 4(b). Student population by major (Y4S2)

The above table and figure show that there are (174) students enrolling at fourth year second semester with biggest student numbers in Civil and Information Technology (IT) engineering majoring.

Based on the analysis, it is distinct that the student population reduce from (257) to (174) after four academic years. It may be because of drop-out or transfer to other universities.

Table 5. Students population trend by major (Y1S1) to (Y4S2)

Major	Civil	Arch	Mech	EC	EP	IT	McE	Che
Y1,S1	40	29	41	41	35	37	20	14
Y4,S2	33	22	23	23	27	29	11	6
Decrease%	18	24	44	44	23	22	45	57
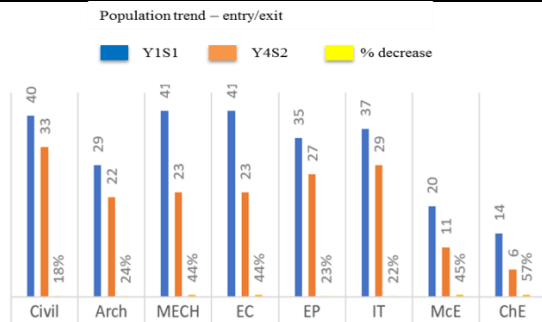
Fig 5. Students population trend by major (Y1S1) to (Y4S2)

In the above figure, it can be interpreted that, in general, student population reduce from first year to fourth year in every major because of many reasons. Of the eight majoring, Electronics, Mechanical and Chemical Engineering Major have significant changes in population.

Table 6. Students mathematical performance by major (Y1S1)

Major	Civil	Arch	Mech	EC	EP	IT	McE	Che
Present	40	29	41	41	35	37	20	14
Excellent	25	11	10	18	25	17	8	2
Excellent%	63	38	24	44	71	46	40	14
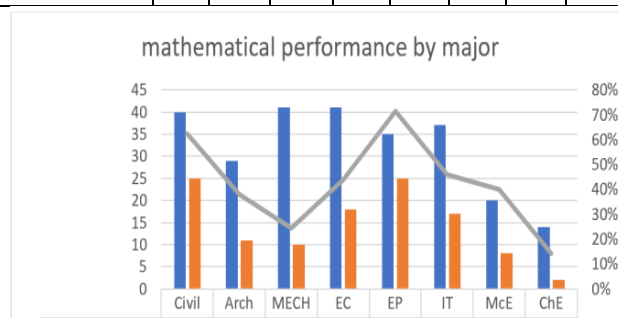
Fig 6. Student mathematical performance by major (Y1S1)

In terms of mathematical performance, the above analysis show that Electrical Power (EP) students have the highest mathematical performance with (71%) of student passed with excellent while chemical engineering with just the above average result only (14%) of students passed with excellent in engineering mathematics.

Table 7. Students' performance trend in civil major for three batches

		1st Year		2nd Year		3rd Year		4sth Year	
		S1	S2	S1	S2	S1	S2	S1	S2
Batch One	Pr	40	38	37	37	35	35	33	33
	E	25	24	28	16	23	26	28	24
Batch Two	Pr	44	43	42	44	41	41	40	40
	E	30	36	30	25	23	19	21	34
Batch Three	Pr	71	71	63	64	60	60	61	61
	E	54	51	52	42	47	53	57	49
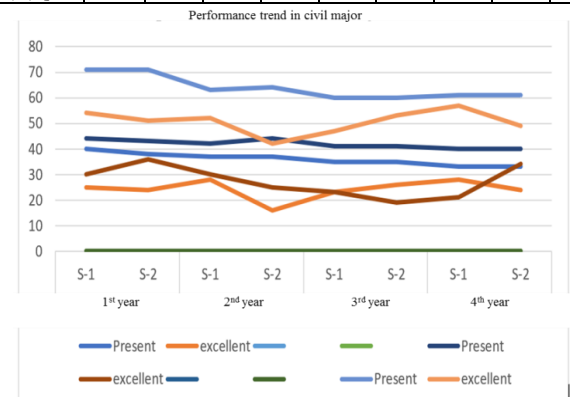
Fig 7. Students' performance trend in civil major for three batches

More detail analysis on mathematical performance of students shall be conducted as in the figure above. It can be highlighted that performance in all second semesters examination result is as good as that in first semesters.

Table 8. Students examination fail trend across three batches

		1st Year		2nd Year		3rd Year		4sth Year	
		S1	S2	S1	S2	S1	S2	S1	S2
Batch One	Pr	238	236	215	206	194	191	173	172
	F	19	15	6	8	2	2	4	2
Batch Two	Pr	250	232	222	220	203	203	195	194
	F	3	10	11	14	11	5	8	9
Batch Three	Pr	312	336	288	287	294	293	296	295
	F	48	30	13	13	2	4	6	8
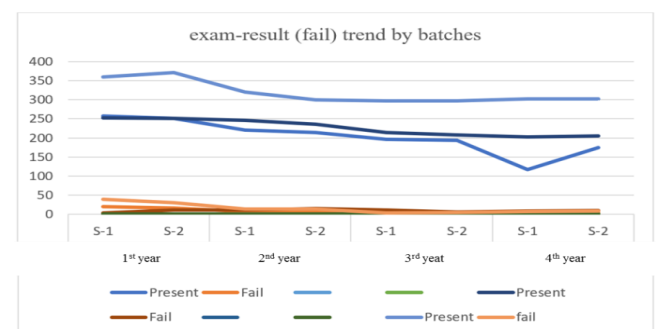
Fig 8. Student examination fail trend across three batches

In terms of examination result (fail) , the above analysis show that with the decrease in enrollment in each batch and semester as the students become more senior, only first and second semester students fail in the mathematics subject significantly while the fail rate decrease as the seniority increase in every majoring.

Table 9. Students population trend by batch (Y1S1) to (Y4S2)

	Batch One	Batch Two	Batch Three	Average
Total In	257	253	360	290
Total Out	174	208	303	228
Retainage	68%	82%	84%	79%
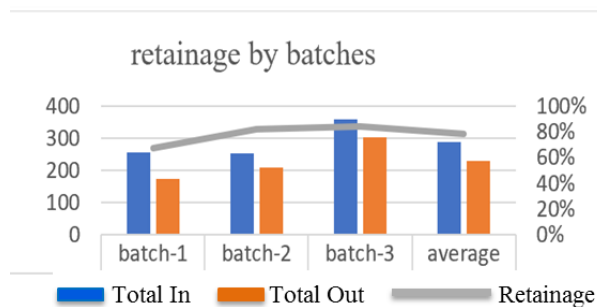
Fig 9. Students population trend by batch (Y1S1) to (Y4S2)

In the above table and figure, based on analysis of student's enrollment and completion of the courses, it is obvious that the number of student reduces in every batch after completion of four year study in engineering mathematics. In terms of percentage reduction, in average, about 20% of students do not complete four year-eight semester long engineering mathematics study while only 80% of original first year first semester enrollment is retained. The following table shows the summary of graduation and drop out for batch one students.

Table 10(a). The distribution of information on the summary sheet by SPSS

	Total In	Total Out	Graduation %	Drop Out %
Batch One	257	174	68	32
Batch Two	253	208	82	18
Batch Three	360	303	84	16

The above table means that the percentage of a student to graduate and drop out for batch one is 68% and 32% respectively. Similarly, the data for batch two and batch three are (82% and 18%) and (84% and 16%) respectively.

This information is very helpful for management decision making in nurturing and programming for engineer requirement for the certain period.

6.3 Absorbing Markov chain

The calculation of data for the Batch One and an Absorbing Markov chain is introduced and the main steps of the Decision Making process and examples are presented to illustrate the applicability of the constructed model to decision making situations of our day to day life [Ni Ni Win (2019)].

From that research, the future performance can be predicted from the current enrolment of students in a four years academic programme. The result for batch one is shown in Table 10(b) as follows:

Table 10(b). The distribution of information on the summary sheet in Markovian matrix

	1st Year		2nd Year		3rd Year		4th Year	
	S-1	S-2	S-1	S-2	S-1	S-2	S-1	S-2
P	238	236	215	206	194	191	173	172
R	19	15	6	8	2	2	4	2
W	0	6	8	7	18	3	13	3
G	0	0	0	0	0	0	0	0
Total	257	257	229	221	214	196	190	177

where P represents Promoted, R represents Repeated, W represents Withdrawn or transfer, and G represents Graduating [Ni Ni Win (2019)]. The following matrix

$$B = \begin{matrix} & s_5 & s_6 \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix} \begin{bmatrix} .251 & .748 \\ .235 & .765 \\ .175 & .825 \\ .085 & .915 \end{bmatrix}.$$

means that Percentage of withdrawal or transfer and graduation of the first year students, second year students, etc.

Further, observing the matrix B, we find that $b_{16} = 0.748$ and it means that the probability of a student to graduate for batch one is 75%.

The probability of a student to graduate for batch one is 75% while the mean time needed for his/her graduation is 4.14 years. But the current enrolment of students in a six years academic programme, a total of 257 students enrolled in first year, 229 in second year, 214 in third year, 190 in fourth year, 174 in fifth and sixth year for the six academic sessions with various categories of graduated, repeated and withdrawn, etc shown in Table 10(b). Therefore, the real results are: the probability of a student to graduate is 68% is a little different from predict solution (74.8%) and other are almost the same while the mean time needed for his/her graduation is 6.14 years.

7. CONCLUSIONS AND RECOMMENDATION

Descriptive analysis on collected compiled data and the corresponding results are presented above. Based on analysis, the number of students studying in each academic year, semesters and in different majoring are significantly not the same and reduces year by year and semester by semester as the students become more and more senior. The more senior in year, the less the number of students. That may be due to the difference in entry score for each academic year and some of the students may have transferred to other universities because of family condition.

It is found that an average of 32 students enroll in the first year of first semester, in Fig. 4(a), and the enrollment reduces to an average of 22 students in second semester of fourth year, in Fig. 4(b), mathematics study while only 80% of original first year first semester enrollment is retained. This information is very helpful for management decision making in nurturing and programming for engineer requirement for the certain period.

The same type of research shall be conducted for exploring influencing factors on academic performance of engineering student for other universities in order that the management of respective universities can improve relevant facilities and support students perusing quality education.

In case of an Absorbing Markov Chain, we study of the corresponding problem for batch one, the probability of a student to graduate is 68% is a little different from predict solution (74.8%) shown in Table 10(b). This information is the same as the result by descriptive statistics shown in Table 10(a). Hence the approval result had been illustrated.

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Approximate Solutions of Initial Value Problems for Ordinary Differential Equations using Adams-Bashforth and Adams-Moulton Methods

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ABSTRACT*:* Firstly, the most popular theorems such as Taylor's Theorem and Newton's divided difference interpolation for solving Ordinary Differential Equations are presented. Next, the multistep methods as the Adams-Bashforth and the Adams-Moulton methods are derived from the fundamental calculus by using Newton's divided interpolation backward difference and discussed in terms of their order. Moreover, it is found that the Adams-Bashforth method (the predictor) is used to predict a first approximation, y_{n+1}^*, and the Adams-Moulton method (the corrector) is used to obtain a corrected approximation, y_{n+1} whereas the Adams-Moulton methods are more efficient and accurate than Adams-Bashforth methods. Finally, the errors in the comparisons of Adams-Bashforth methods and Adams-Moulton methods for the first order initial value problems are illustrated with examples.

KEYWORDS: *Adams-Bashforth method, Adams-Moulton method, backward difference formula, interpolation polynomial, initial value problem.*

1. INTRODUCTION

The differential equations are one of the most important mathematical tools used in modelling problems in physical sciences. In this paper, we derive and analyze some numerical multistep methods for solving problems for ordinary differential equations. The main form of the problem that we study is the initial value problem:

$$y' = f(x,\ y),\qquad y(x_0) = y_0.$$

The function $f(x,y)$ is to be continuous for all (x,y) in some domain D of the *xy*-plane and (x_0, y_0) is a point in D. Taylor methods and Runge-Kutta(RK) methods are known as *single-step* or *one-step methods*, since at a typical step is y_{n+1} is determined solely from y_n. It is considered that the multistep methods in which the computation of the numerical solution y_{n+1} uses the solution values at several previous nodes. There are many such multistep methods, however, it will be considered the only most popular of them, two families of the Adams-Bashforth (AB) and Adams-Moulton (AM) methods. These methods are the basis of some of the most widely used computer codes for solving the initial value problem in [Butcher (2000)], [Feldsten & Stetter (1963)], [Adesanya, Odekunle & Alkali (2012)]. Standard introductory texts are [Hoffman (2001)],

[Scott (2011)] and [Stoer & Bulirsch (1993)]. [Keryszig (2011)], [Butcher (2000)] and [Adesanya, Odekunle & Alkali (2012)] give the details on some theoretical issues, although the stability of multistep methods are not discussed. Furthermore, [Singh (2014)], [Kendall, Weimin, & David (2009)] and [Yang, Wenwu, Chung, & Morris (2005)] give a more complete recent survey of the field.

The most widely utilized linear multistep methods used for nonstiff problems are the Adams - Bashforth methods, which are members of the Adams Family that are explicit. An alternative family of implicit linear multistep methods is the family of backwards differentiation formulas, which are the topic of the next section**.**

2. MULTISTEP METHODS

Taylor's theorem for functions and Newton's divided difference interpolation for approximating functions $f(x)$ by polynomials are presented in this section. Taylor's theorem gives a relatively simple method for approximating functions $f(x)$ by the polynomials and thereby gives a method for computing $f(x)$.

2.1 Taylor's Theorem

Let $f(x)$ have $n+1$ continuous derivatives on [a, b] for some $n \ge 0$ and let $x, x_0 \in [a, b]$. Then
$$f(x) = p_n(x) + R_{n+1}(x) \qquad (1)$$

$$p_n(x) = f(x_0) + \frac{(x-x_0)}{1!}f'(x_0) + \ldots + \frac{(x-x_0)^n}{n!}f^{(n)}(x_0),$$

$$R_{n+1}(x) = \frac{1}{n!}\int_{x_0}^{x}(x-t)^n f^{(n+1)}(t)\,dt = \frac{(x-x_0)^{n+1}}{(n+1)!}f^{(n+1)}(\xi)$$

for some ξ between x_0 and x [Feldsten & Stetter (1963)], [Keryszig (2011)].

2.2 Newton's Divided Difference Interpolation

For interpolating the value of the function $y = f(x)$ near the values and to extrapolate value of the function a short distance forward from y_n, Newton's backward interpolation formula is used. Let $y = f(x)$ be a function which takes on values $f(x_n)$, $f(x_n - h)$, $f(x_n - 2h), \ldots, f(x_0)$ corresponding to equidistant values $x_n, x_n - h, x_n - 2h, \ldots, x_0$. Suppose we have

to evaluate the function $f(x)$ at $(x_n + rh)$, where r is any real number, then we have the shift operator E, such that

$$f(x_n + rh) = E^r f(x_n) = (E^{-1})^{-r} f(x_n) = (1 - \nabla)^{-r} f(x_n)$$

Binomial expansion yields,

$$f(x_n + rh) = \left[1 + r\nabla + \frac{r(r+1)}{2!}\nabla^2 + \frac{r(r+1)(r+2)}{3!}\nabla^3 + \dots \right.$$

$$\left. + \frac{r(r+1)(r+2)\dots(r+n-1)}{n!}\nabla^n + \text{Error} \right] f(x_n)$$

That is

$$f(x_n + rh) = f(x_n) + r\nabla f(x_n) + \frac{r(r+1)}{2!}\nabla^2 f(x_n) + \dots$$

$$+ \frac{r(r+1)(r+2)\dots(r+n-1)}{n!}\nabla^n f(x_n) + \text{Error}.$$

This formula is known as Newton's-Gregory backward difference interpolation formula.

If we retain $(r+1)$ terms, we obtain a polynomial of degree r agreeing with $f(x)$ at $x_n,\ x_{n-1},\ \dots,\ x_{n-r}$. Alternatively, this formula can also be written as

$$y_x = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n + \dots$$

$$+ \frac{r(r+1)(r+2)\dots(r+n-1)}{n!}\nabla^n y_n + \text{Error}.$$

Here $r = \dfrac{x - x_n}{h}$.

The results obtained for (1) will generalize in a straightforward way to both systems of differential equations and higher order equations, provided appropriate vector and matrix notation which are used.

These generalizations are discussed and illustrated in the next two sections. Now, we consider the multistep method which is more accurate than one-step method, [Feldsten & Stetter (1963)], [Hoffman (2001)] and [Stoer & Bulirsch (1993)].

A predictor-corrector method is a set of two equations for y_{n+1}. In general, the corrector depends on the predicted value. The reason for using the multistep approach is that the additional information might increase accuracy. Such methods are obtained as follows.

3. DERIVATION OF MULTISTEP METHODS

Consider an initial value problem in the equation $Y'(x) = f(x, Y(x))$, $Y(x_0) = Y_0$ as before with f such that the problem has a unique solution on some interval containing x_0. We integrate $Y'(x) = f(x, Y)$, from x_n to $x_{n+1} = x_n + h$. This gives

$$\int_{x_n}^{x_{n+1}} Y'(x)\, dx = \int_{x_n}^{x_{n+1}} f(x, Y(x))\, dx,$$

$$Y(x_{n+1}) = Y(x_n) + \int_{x_n}^{x_{n+1}} f(x, Y(x))\, dx. \qquad (2)$$

We will develop numerical methods to compute the solution $Y(x)$ by approximating the integral in (2). They are generally more efficient than the Runge-Kutta methods, especially if one can find the solution with a high degree of accuracy or if the derivative function $f(x, Y)$ is expensive to evaluate.

To evaluate the integral $\int_{x_n}^{x_{n+1}} g(x)\, dx$,

$$g(x) = Y'(x) = f(x, Y(x)), \qquad (3)$$

we approximate $g(x)$ by using polynomial interpolation and then integrate the interpolating polynomial. For a given nonnegative integer q, the AB methods use interpolation polynomial of degree q at the points $\{x_n,\ x_{n-1},\ \dots,\ x_{n-q}\}$ and AM methods use interpolation polynomial of degree q at the points $\{x_{n+1},\ x_n,\ x_{n-1},\ \dots,\ x_{n-q+1}\}$.

Therefore we will use the interpolating polynomials with backward difference formulas as p_n for AB methods and $\tilde{p}_n$ for AM methods.

$$p_n(x) = f_n + r\nabla f_n + \frac{r(r+1)}{2!}\nabla^2 f_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 f_n + \dots$$

$$+ \frac{r(r+1)(r+2)\dots(r+n-1)}{n!}\nabla^n f_n \qquad (4)$$

where $r = \dfrac{x - x_n}{h}$.

$$\tilde{p}_n(x) = f_{n+1} + r\nabla f_{n+1} + \frac{r(r+1)}{2!}\nabla^2 f_{n+1} + \frac{r(r+1)(r+2)}{3!}\nabla^3 f_{n+1}$$

$$+ \dots + \frac{r(r+1)(r+2)\dots(r+n-1)}{n!}\nabla^n f_{n+1} \qquad (5)$$

where $r = \dfrac{x - x_{n+1}}{h}$.

It is practical to replace these differences by their expressions in terms of f:

$$\nabla f_n = f_n - f_{n-1},$$

$$\nabla^2 f_n = f_n - 2f_{n-1} + f_{n-2},$$

$$\nabla^3 f_n = f_n - 3f_{n-1} + 3f_{n-2} - f_{n-3},\ \dots.$$

$$\nabla f_{n+1} = f_{n+1} - f_n,$$

$$\nabla^2 f_{n+1} = f_{n+1} - 2f_n + f_{n-1},$$

$$\nabla^3 f_{n+1} = f_{n+1} - 3f_n + 3f_{n-1} - f_{n-2},\ \dots.$$

We substitute these into the polynomials p_n and collect terms and we then obtain the multistep formula of the AB methods and AM methods of any order, [Feldsten & Stetter (1963)], [Hoffman (2001)], [Keryszig (2011)] and [Scott (2011)].

3.1 Adams-Bashforth Methods

We begin with the AB method based on linear interpolation (q = 1). The linear polynomial interpolating $Y(x)$ at $\{x_n,\ x_{n-1}\}$ is

$$p_1(x) = f_n + r\nabla f_n. \qquad (6)$$

We replace f(x, $Y(x)$) by an interpolation polynomial $p_1(x)$ so that we can later integrate.

This gives approximations Y_{n+1} of $Y(x_{n+1})$ and Y_n of $Y(x_n)$.

$$Y_{n+1} = Y_n + \int_{x_n}^{x_{n+1}} p_1(x)\,dx. \qquad (7)$$

Different choice of p(x) will now produce different methods. We explain the principle by taking a polynomial, $p_1(x)$ that at (equidistant) x_n, x_{n-1} has the respective values

$$f_n = f(x_n, Y_n), \ f_{n-1} = f(x_{n-1}, Y_{n-1}).$$

This will lead to a practically usefully formula. We can obtain $p_1(x)$ from the Newton backward difference formula in (6) with $rh = (x - x_n)$.

We integrate $p_1(x)$ over x from x_n to $x_{n+1} = x_n + h$, thus over r from 0 to 1. Since $x = x_n + hr$, we have $dx = hdr$. Thus we obtain

$$Y_{n+1} = Y_n + h[f_n r + \frac{r^2}{2}\nabla f_n]\Big|_0^1 = Y_n + h[f_n + \frac{1}{2}\nabla f_n]$$

$$= Y_n + h[f_n + \frac{1}{2}(f_n - f_{n-1})]$$

$$Y_{n+1} = Y_n + \frac{h}{2}[3f_n - f_{n-1}]. \qquad (8)$$

Higher order Adams-Bashforth methods are obtained by using higher degree polynomial interpolation in the approximation of the integrand in (4). So, in this paper, the next higher order methods are computed and shown in the following table.

Table 1. Adams-Bashforth Methods

q	Order	Methods
0	1	$Y_{n+1} = Y_n + hf_n$
1	2	$Y_{n+1} = Y_n + \frac{h}{2}[3f_n - f_{n-1}]$
2	3	$Y_{n+1} = Y_n + \frac{h}{12}[23f_n - 16f_{n-1} + 5f_{n-2}]$
3	4	$Y_{n+1} = Y_n + \frac{h}{24}[55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}]$

The local truncation error is of order h^2, h^3, h^4, h^5, as can be shown, so that the global error is of order h^2, h^3, h^4, hence the methods in Table (1) do define AB *methods* of order 1, 2, 3 and 4 [Feldsten & Stetter (1963)], [Hoffman (2001)], [Keryszig (2011)], [Scott (2011)] and [Stoer & Bulirsch (1993)].

3.2 Adams-Moulton Methods

As with the AB methods, we begin with our presentation of AM methods by considering the method based on linear interpolation of the functions $f(x, Y(x))$ at x_{n+1}, x_n, x_{n-1}, x_{n-2}, $\ldots$, in (5). It is explained that the principle for the polynomial $\tilde{p}_1(x)$ which interpolates at x_{n+1}, x_n. (Before we had x_n, x_{n-1}). By using in the preceding section again and setting $rh = (x - x_{n+1})$, we have

$$\tilde{p}_1(x) = f_{n+1} + r\nabla f_{n+1}. \qquad (9)$$

We integrate over x from x_n to x_{n+1} as before. This now corresponds to integrating over r from −1 to 0. We obtain

$$\int_{x_n}^{x_{n+1}} \tilde{p}_1(x)\,dx = \int_{-1}^{0} (f_{n+1} - r\nabla f_{n+1})h\,dr$$

$$= h[f_{n+1}r + \frac{r^2}{2}\nabla f_{n+1}]\Big|_{-1}^0 = h[f_{n+1} - \frac{1}{2}\nabla f_{n+1}]$$

$$= h[f_{n+1} - \frac{1}{2}(f_{n+1} - f_n)]$$

$$\int_{x_n}^{x_{n+1}} \tilde{p}_1(x)\,dx = \frac{h}{2}[f_{n+1} + f_n]. \qquad (10)$$

Replacing the differences as before gives

$$Y_{n+1} = Y_n + \int_{x_n}^{x_{n+1}} \tilde{p}_1(x)\,dx$$

$$Y_{n+1} = Y_n + \frac{h}{2}[f_{n+1} + f_n]. \qquad (11)$$

This is usually called as Adams-Moulton formula of order 2. It is an implicit formula because $f_{n+1} = f(x_{n+1}, Y_{n+1})$ appear on the right, so that it defines Y_{n+1} only implicitly, in contrast to (7), which is an explicit formula, not involving Y_{n+1} on the right. To use (8) we must predict a value Y_{n+1}^* for instance, by using backward difference formula, that is,

$$Y_{n+1}^* = Y_n + \frac{h}{2}[3f_n - f_{n-1}]. \qquad (12)$$

The corrected new value Y_{n+1} is then obtained from (8) with f_{n+1} replace by $f_{n+1}^* = f(x_{n+1}, Y_{n+1}^*)$ and the other f's as in (8); thus,

$$Y_{n+1} = Y_n + \frac{h}{2}[f_{n+1}^* + f_n]. \qquad (13)$$

This predictor-corrector method (12), (13) is often called the Adams-Moulton method of second order. Sometimes this name is reserved for the corresponding method with several corrections per step by (10) unit which reaches a specified accuracy.

Higher order Adams-Moulton methods are obtained by using higher degree polynomial interpolation in the approximation of the integrand in (5). Here, the next higher order methods are computed and shown in the following table.

Table 2. Adams-Moulton Methods

q	Order	Methods
0	1	$Y_{n+1} = Y_n + hf_{n+1}^*$
1	2	$Y_{n+1} = Y_n + \frac{h}{2}[f_{n+1}^* + f_n]$
2	3	$Y_{n+1} = Y_n + \frac{h}{12}[5f_{n+1}^* + 8f_n - f_{n-1}]$
3	4	$Y_{n+1} = Y_n + \frac{h}{24}[9f_{n+1}^* + 19f_n - 5f_{n-1} + f_{n-2}]$

Adams methods are often considered to be less expensive than Runge-Kutta (RK) methods and the main reason can be observed by comparing to the

Adams formulas with the corresponding order RK method. The main task of both methods is to evaluate the derivative function $f(x, Y)$.

Thus, according to Adams methods, we must first compute $Y_1, Y_2, Y_3, \dots$ by some other methods of great accuracy, for instance, by a Runge-Kutta method or a Runge-Kutta-Fehlberg method. Note that if f depends only on x, this method reduces to Simpson's rule of integration. Therefore, it is presented as the definition of RK methods.

The general family of r-stage explicit Runge-Kutta methods is given by

$$Y_{n+1} = Y_n + h\sum_{i=1}^{r} b_i k_i, \qquad (14)$$

where $\quad k_1 = f(x_n, Y_n)$

$k_2 = f(x_n + c_2 h, Y_n + h a_{21} k_1(x_n, Y_n)),$

$k_3 = f(x_n + c_3 h, Y_n + h(a_{31} k_1(x_n, Y_n) + a_{32} k_1(x_n, Y_n))),$

$\vdots$

$$k_r = f(x_n + c_r h, Y_n + h\sum_{j=1}^{r} a_{rj} k_j).$$

To specify a particular method, we need to provide the integer r (the number of stages), and the coefficients $c_i (i = 2, 3, \dots, r)$, $a_{ij} (1 \le j < i \le r)$, and $b_i (i = 1, 2, \dots, r)$.

These data are usually arranged in a co-called *Butcher tableau* (after John C. Butcher):

$$
\begin{array}{c|ccccc}
0 & & & & & \\
c_2 & a_{21} & & & & \\
c_3 & a_{31} & a_{32} & & & \\
\vdots & \vdots & \vdots & \ddots & & \\
\vdots & \vdots & \vdots & & & \\
c_r & a_{r1} & a_{r2} & & a_{r,r-1} & \\
\hline
 & b_1 & b_2 & \dots & b_{r-1} & b_r
\end{array}
$$

Now we will present the approximations of the initial value problems by using these methods and compare the accuracy of three multistep methods by means of example, [Feldsten & Stetter (1963)], [Hoffman (2001)], [Keryszig (2011)], [Scott (2011)] and [Stoer & Bulirsch (1993)].

3.3. Example

Consider the initial-value problem $y' = (y - x - 1)^2 + 2$, with the initial condition $y(0) = 1$. The exact solution of this IVP is easily evaluated and so is $(\tan x + x + 1)$. Before computing the multistep methods, the initial approximations are evaluated by Runge-Kutta methods for $h = 0.05$ and 0.1. It is computed by applying the AB methods and AM methods for order 2, 3 and 4 with $h = 0.05$ and $h = 0.1$ for time-steps $0 \le x \le 0.5$ and $0 \le x \le 1$, the error comparison of its exact solution and the approximate solutions are given as the following tables and figures respectively.

Table 3. Comparison of AB2, AB3, AB4 and Exact Solution for $y' = (y - x - 1)^2 + 2$, $h = 0.05$

n	x	Exact	AB2	AB3	AB4	Error AB2	Error AB3	Error AB4
1	0.05	1.100 0417						
2	0.1	1.200 3347	1.200 4394			-0.000 1047		
3	0.15	1.301 1352	1.301 2607	1.301 1325		-0.000 1255	2.7E-06	
4	0.2	1.402 7100	1.402 8562	1.402 7057	1.402 7081	-0.000 1462	4.3E-06	-1E-07
5	0.25	1.505 3419	1.505 5094	1.505 3360	1.505 3400	-0.000 1675	5.9E-06	-3E-07
6	0.3	1.609 3362	1.609 5260	1.609 3288	1.609 3342	-0.000 1898	7.4E-06	-5E-07
7	0.35	1.715 0284	1.715 2421	1.715 0193	1.715 0262	-0.000 2137	9.1E-06	-8E-07
8	0.4	1.822 7932	1.823 0330	1.822 7820	1.822 7905	-0.000 2398	1.12 E-05	-1E-06
9	0.45	1.933 0551	1.933 3240	1.933 0416	1.933 0519	-0.000 2689	1.35 E-05	-1.3E-06
10	0.5	2.046 3025	2.046 6044	2.046 2862	2.046 2986	-0.000 3019	1.63 E-05	-1.8E-06

Table 4. Comparison of AM2, AM3, AM4 and Exact Solution for $y' = (y - x - 1)^2 + 2$, $h = 0.05$

| n	x	Exact	AM2	AM3	AM4	Error AM2	Error AM3	Error AM4
1	0.05	1.100 0417						
2	0.1	1.200 3347	1.200 5653			-0.000 2306		
3	0.15	1.301 1352	1.301 3902	1.301 1325		-0.000 2550	2.7E-06	
4	0.2	1.402 7100	1.402 9919	1.402 7111	1.402 7101	-0.000 2819	-1.1E-06	-1E-07
5	0.25	1.505 3419	1.505 6542	1.505 344	1.505 3422	-0.000 3123	-2.1E-06	-3E-07
6	0.3	1.609 3362	1.609 6833	1.609 3395	1.609 3367	-0.000 3471	-3.3E-06	-5E-07
7	0.35	1.715 0284	1.715 4159	1.715 0333	1.715 0292	-0.000 3875	-4.9E-06	-8E-07
8	0.4	1.822 7932	1.823 2281	1.822 8001	1.822 7942	-0.000 4349	-6.9E-06	-1E-06
9	0.45	1.933 0551	1.933 5462	1.933 0646	1.933 0564	-0.000 4911	-9.5E-06	-1.3E-06
10	0.5	2.046 3025	2.046 8610	2.046 3154	2.046 3043	-0.000 5585	-1.29 E-05	-1.8E-06
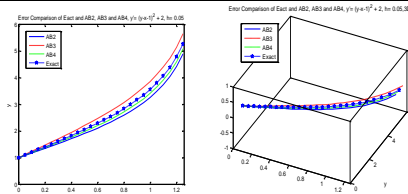
Fig 1. Error comparison between AB approximations and exact solution for $y' = (y - x - 1)^2 + 2$, $h = 0.05$ with (a) 2D and (b) 3D
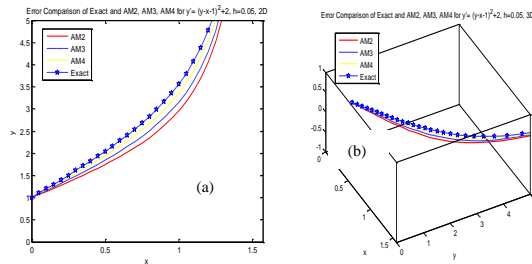
Fig 2. Error comparison between AM approximations and exact solution for $y' = (y-x-1)^2 + 2$, $h = 0.05$ with (a) 2D and (b) 3D

Fig 3. Error comparison between AB approximations and exact solution for (a) $= (y-$ (b) $^2 + 2$, $h = 0.1$ with (a) 2D and (b)3D

Table 5. Comparison of AB2, AB3, AB4 and Exact Solution for $y' = (y-x-1)^2 + 2$, h = 0.1

n	x	Exact	AB2	AB3	AB4	Error AB2	Error AB3	Error AB4
1	0.1	1.2003347						
2	0.2	1.4027100	1.4035606			-0.0008506		
3	0.3	1.6093362	1.6103505	1.6108233		-0.0010143	-0.0014871	
4	0.4	1.8227932	1.8239628	1.8242982	1.8227151	-0.0011696	-0.001505	7.81E-05
5	0.5	2.0463025	2.0517033	2.0478927	2.0461974	-0.0054008	-0.0015902	0.0001051
6	0.6	2.284137	2.2956949	2.2858257	2.2839783	-0.0115579	-0.0016887	0.0001587
7	0.7	2.5422884	2.5627914	2.5440799	2.5420273	-0.020503	-0.0017915	0.0002611
8	0.8	2.8296386	2.8632496	2.8314883	2.8291712	-0.033611	-0.0018497	0.0004674
9	0.9	3.1601582	3.2134701	3.161883	3.1592473	-0.0533119	-0.0017248	0.0009109
10	1	3.5574077	3.6415751	3.5584099	3.5554514	-0.0841674	-0.0010022	0.0019563

Table 6. Comparison of AM2, AM3, AM4 and Exact Solution for $y' = (y-x-1)^2 + 2$, h = 0.1

n	x	Exact	AM2	AM3	AM4	Error AM2	Error AM3	Error AM4
1	0.1	1.2003347						
2	0.2	1.4027100	1.4045921			-0.0018821		
3	0.3	1.6093362	1.6115008	1.6109069		-0.0021646	-0.0015707	
4	0.4	1.8227932	1.8289546	1.8245049	1.8227981	-0.0061614	-0.0017117	-4.9E-06
5	0.5	2.0463025	2.0581632	2.0482207	2.0463149	-0.0118607	-0.0019182	1.24E-05
6	0.6	2.284137	2.3040216	2.2863536	2.284161	-0.0198846	-0.0022166	-2.4E-05
7	0.7	2.5422884	2.5735647	2.5449407	2.5423319	-0.0312763	-0.0026523	4.35E-05
8	0.8	2.8296386	2.8774811	2.8329421	2.8297142	-0.0478425	-0.0033035	7.56E-05
9	0.9	3.1601582	3.2330643	3.1644719	3.160288	-0.0729061	-0.0043137	-0.0001298
10	1	3.5574077	3.6704865	3.5633665	3.5576256	-0.1130788	-0.0059588	-0.0002179
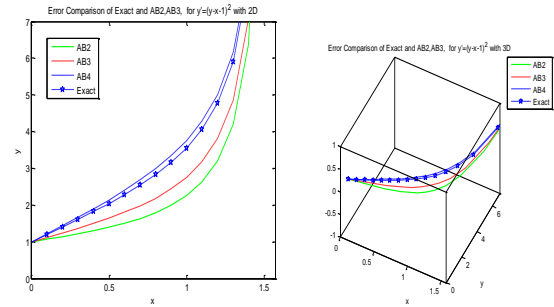
Fig 4. Error comparison between AM approximations and exact solution for $y' = (y-x-1)^2 + 2$, $h = 0.1$ with (a) 2D and (b) 3D

From tables, it is observed that the AM3 method is more accurate than AM2 method and AM4 method gives more accuracy than the two methods. Then it is discovered that the approximate solutions are closed to the exact solutions for using step size $h = 0.05$.

4. CONCLUSIONS

Although other factors affect the choice of a numerical method, the AB methods and AM methods are generally more efficient in the number of evaluations of f that are needed for a given amount of accuracy. It is sufficient for our purposes to illustrate the idea of a predictor-corrector method using the simplest possible pair of methods. The acquired results are displayed in Tables (3-6) and graphically presented in Figures (1-4) by using MATLAB. The approximate solutions and errors are calculated with the step sizes

0.05, and 0.1. Besides, the approximate solutions are compared to the exact solutions. From the resulting tables and figures, it is analyzed that three AB methods and AM methods converge to the exact solutions for the step size, h. The fourth order approximations for same step size converges to exact solution. It is also mentioned that the small step size provides the better approximation. From the numerical example, it is shown clearly that higher order AB methods is more appropriate than lower order AB methods to find more accurate result. Moreover, it is also concluded that higher order AM methods is better in term of accuracy than lower order AM methods for the first order initial value problems.

Moreover, we have to solve the population model and the chemical reaction model of ordinary differential equations by AB and AM methods as future work.

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Effective Queue Management of Three Supermarkets in Myeik City

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ABSTRACT*:* The purpose of this paper is to identify effective and effective anti-queuing forces by using queuing theory. In the first part, the concerns of queuing theory are introduced. And then the basic definitions of queuing theory such as queues, queuing system with illustrated figures, traffic intensity and terminology of queuing system are presented. Finally, the traffic intensity, the expected number of customers, the expected number of customers waiting in the queue, expected waiting time of a customer in the system.

KEYWORDS: *Queuing, Effective anti-queuing, Traffic intensity, Terminology, Expected number, Waiting time*

1. INTRODUCTION

A queuing system is essential when there are other people or services that require a service from another organization. There is a service and a person has a line or a queue in the system. The main point of this paper is relation to the pending events of Myeik city, which is the modern division of Thanintharyi. The important facts are the major costs associated with queuing process. In most cases, research and experience show that customers will often leave a store without purchasing. Rather than buying a stand or slow moving line. Although there is a huge improvement in technology designed to reduce waiting times, every management is a challenge for retailers and shop owners [Anthony Igwe, 2014]. The truth is, if queues are not managed properly, they are not satisfied with customers. Sales decline will result in customer satisfaction and loss of customer loyalty. In particular, owners, retailers, are trying to measure and manage long-term waiting plans. Basically, many methods have been used. Some are expensive, exit counters, unscrupulous solutions as a commercial solution, commercial software and expensive hardware package that include mechanical engineering studies and clients satisfaction survey. But despite these efforts, today's retailers are still dissatisfied with them shopping experience [XING Wenjie, 2015].

2. COMPONENTS OF QUEUING THEORY

In this section, the fundamental concepts and formulas are presented.

2.1 Queues

Queues are interesting applications of probability. Everyone is waiting in line such as waiting at a check-out counter, try to call over again by telephone, waiting for the time to travel to the gate on the highway was common computer to run our program, etc. the list is almost endless, and the people can quickly and several more examples from his or her own experiences. Technically, queuing is important for the following reasons:

1. As a user's perspective, the queue will be frustrating and will endure only at the least level of inconvenience.
2. Queues are needed from a service provider's perspective and many services cannot provide only the fastest possible speed.

2.2 Queuing System

A queuing system can be described by, the arrival input, the service mechanism, the queue discipline, customer's behavior, [S.Sanmugasundarran, 2015]. The arrival input indicates the joining to the customer system. The mean arrival is denoted by λ. The service mechanism means the arrangement of service facility to serve customers. One of the mathematical distributions often used to describe service times distribution is the negative exponential distribution. The mean service rate is denoted by μ.

2.3 The Queue Discipline

The most common disciplines are first come first served (FCFS), last come first served (LCFS), etc. Low-cost service is given priority to customers in the advance protection system. In the pre-emptive system, a lower level of service is provided prior to performing a service to the client.

2.4 Customer's Behavior

The customers generally act in the following four ways [Banks, pp 24-37], [XING Wenjie, 2015]:

Balking - A customer who leaves the queue because there is no time to wait or there is not enough waiting space.

Reneging – This happens when a customer who waits for impatience leaves the queue.

Priorities – In some applications, the customer is not related to their arrival. This customer has to prioritize others.

Jockeying – Customers can move from one waiting line to the next. This is most common in a supermarket.

2.5 Operating Characteristics of Queuing System

The characteristics of the operational of the queuing system to evaluate the performance of an

existing queuing system and to design a new system are listed below [Banks, pp 24-37], [XING Wenjie, 2015].

Expected waiting time in system: It is the average amount of time represented by the customer in the system. Generally, it is considered a time of waiting and service. It can be used to compare business with other queuing systems.

Expected waiting time in queuing: The average customer spends time on the queue before the comment of his or her service, and can be used to evaluate service quality.

Expected number of customers in the system: This is the number of customers whether they are waiting or receiving services. It can be used to find out when the user is using the system.

Expected number of customers in the queue (queue length): It is the number of customers waiting to be serviced.

The server utilization factor (or busy period): It is the proportion of the time that a server actually spends with the customers. It gives an idea of the expected amount of ideal time which can be used for some other work not directly involved with service.

2.6 Transient State and Steady States

When its characteristics depend on time, it is said that a system is in a transient state. A steady state is independent of time system behavior. Let $P_n(t)$ denote the probability that there are *n* customers in the system at time t. Then in steady state

$$\lim_{t\to\infty} P_n(t) = P_n \text{ (independent of t)}$$

(or) $\dfrac{dP_n(t)}{dt} = \dfrac{dP_n}{dt}$

(or) $\lim_{t\to\infty} P_n'(t) = 0$

2.7 Traffic Intensity (or Utilization Factor)

The expected fraction of time, the individual servers are busy.

$$Traffic\ intensity = \rho = \frac{Mean\ arrival\ rate}{Mean\ service\ rate}$$

$$= \frac{\lambda}{\mu}$$

The unit of traffic intensity is *Erlang.*

2.8 Basic Notations and Formulas

The terms used for analyzing a queuing system are as follows:

$$P_n = \rho^n(1-\rho)$$

$$\frac{\lambda}{\mu} = \rho = \frac{1/\mu}{1/\lambda} \qquad (1)$$

$$L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} = \lambda W_s \qquad (2)$$

$$L_q = L_s - \frac{\lambda}{\mu} \qquad (3)$$

$$W_s = \frac{1}{\mu-\lambda} \qquad (4)$$

$$W_q = W_s - \frac{1}{\mu} \qquad (5)$$

n = the total number of customers

P_n = probability of exactly n customers in queuing system at time t, given number at time zero.

λ = arrival rate

μ = service rate

S = number of service channels

N = maximum number of customer allowed in the system

L_s = Expected number of customers in the system

L_q = Expected number of customers waiting in the queue

W_s = Expected waiting time of a customer in the system

W_q = Expected waiting time of a customer in the queue

2.9 Classification of Queuing Models

Queuing models are also called waiting line [S.Sanmugasundarran, 2015]. The queuing models are classified as follows:
1) Model I: (M/M/1): (∞/FCFS)
2) Model II: Multiservice Model (M/M/S): (∞/FCFS)
3) Model III: (M/M/1): (N/FCFS)
4) Model IV: (M/M/S): (N/FCFS)

The input source is the customer who needs the service over time and the service mechanism, who gives the necessary services for the customers.
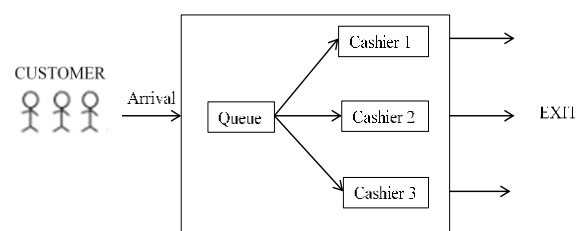
Fig 1. Single Stage Queuing Model with Single-

Queue and Multiple Parallel Servers
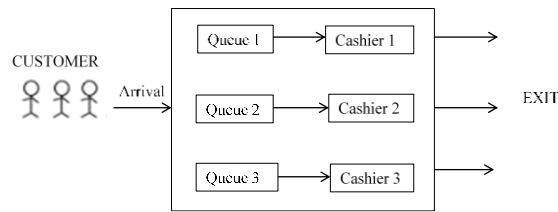
Fig 2. Single Stage Queuing Model with Multiple Queues and Multiple Parallel Servers

3. APPLICATION OF QUEUING THEORY IN MARKETING

This section is the main aim of this paper, the problem is constructed with a market researcher and the three top super markets in Myeik city as follow.

A market researcher investigated the three Supermarkets on the city of Myeik. As the results of her research, the information is provided with the follow Table 1.

Table 1. Supermarkets Data Concerning the Mean Arrival and Mean Service

No	Name of Supermarket	Mean Arrival per Hour	Mean Service per Hour
1	Myeik Shopping Mall	12	15
2	Market Garden	10	12
3	Grand Jade	7	9

According to the data, the average hourly rate for three supermarkets is 7-12. Myeik Shopping Mall has the highest average reach and service of 12 and 15 respectively. Myeik Shopping Mall is located by the sea side and has a great sea view due to its great seating potential. There are a large number of buyers who come to visit and it seemed to enjoy the most strategic location to compare other supermarkets and very close to Market Garden Supermarket. In the Market Garden, in terms of mean arrival and mean service per hour, are 10 and 12 respectively. Its location is a better thing compared to Grand Jade. According to the study, Myeik Shopping Mall is the highest service rate per hour. The difference of mean service per hour between Myeik Shopping Mall and Grand Jade is 6. This is only 9 because of low usage of services customers can serve at an hourly rate. The Grand Jade is located in the center of town and has a few customers because of the difficulty of parking for customer.

The objective of researcher is to give the following:

For Myeik Shopping Mall

Mean arrival per Hour $= \lambda = 12$

Mean service per Hour $= \mu = 15$

Traffic Intensity $= \rho = \dfrac{\lambda}{\mu} = \dfrac{12}{15} = 0.80$

Expected number of customers in the system

$= L_s = \dfrac{\lambda}{\mu - \lambda} = \dfrac{12}{15 - 12} = 4.00$

Expected number of customers waiting in the queue

$= L_q = L_s - \dfrac{\lambda}{\mu} = 4.00 - 0.80 = 3.20$

Expected waiting time of a customer in the system

$= W_s = \dfrac{1}{\mu - \lambda} = \dfrac{1}{15 - 12}$

$= \dfrac{1}{3} = 0.33$

Expected waiting time of a customer in the queue

$= W_q = W_s - \dfrac{1}{\mu} = \dfrac{1}{3} - \dfrac{1}{15}$

$= \dfrac{4}{15} = 0.267$

For Market Garden,

Mean arrival per Hour $= \lambda = 10$

Mean service per Hour $= \mu = 12$

Traffic Intensity $= \rho = \dfrac{\lambda}{\mu} = \dfrac{10}{12} = 0.83$

Expected number of customers in the system

$= L_s = \dfrac{\lambda}{\mu - \lambda} = \dfrac{10}{12 - 10} = 5.00$

Expected number of customers waiting in the queue

$= L_q = L_s - \dfrac{\lambda}{\mu} = 5.00 - \dfrac{10}{12} = 4.17$

Expected waiting time of a customer in the system

$= W_s = \dfrac{1}{\mu - \lambda} = \dfrac{1}{12 - 10}$

$= \dfrac{1}{2} = 0.50$

Expected waiting time of a customer in the queue

$= W_q = W_s - \dfrac{1}{\mu} = \dfrac{1}{2} - \dfrac{1}{12}$

$= \dfrac{5}{12} = 0.417$

For Grand Jade

Mean arrival per Hour $= \lambda = 7$

Mean service per Hour $= \mu = 9$

Traffic Intensity $= \rho = \dfrac{\lambda}{\mu} = \dfrac{7}{9} = 0.78$

Expected number of customers in the system

$$= L_s = \dfrac{\lambda}{\mu - \lambda} = \dfrac{7}{9-7} = 3.50$$

Expected number of customers waiting in the queue

$$= L_q = L_s - \dfrac{\lambda}{\mu} = 3.50 - \dfrac{7}{9}$$

$$= \dfrac{49}{18} = 2.72$$

Expected waiting time of a customer in the system

$$= W_s = \dfrac{1}{\mu - \lambda} = \dfrac{1}{9-7}$$

$$= \dfrac{1}{2} = 0.50$$

Expected waiting time of a customer in the queue

$$= W_q = W_s - \dfrac{1}{\mu} = \dfrac{1}{2} - \dfrac{1}{9}$$

$$= \dfrac{7}{18} = 0.39$$

Hence, a brief results are as shown in Table (2).

Table 2. Computed Values of Supermarkets Data

No	Supermarket	ρ	L_s	L_q	W_s	W_q
1	Myeik Shopping Mall	0.80	4.00	3.20	0.33	0.267
2	Market Garden	0.83	5.00	4.17	0.50	0.417
3	Grand Jade	0.78	3.50	2.72	0.50	0.39

The data in table 2 displays the calculated values of the equations (1) to (5). Equation (3) shows that the average numbers of customers in the queue of three supermarkets are 3.20, 4.17, and 2.72. As equation (2), the average number of customers in the system is generally gravitated around 4.2. The average time spent waiting for the service is as shown in equation (5) as the customers time spent of three supermarkets are 0.267, 0.417 and 0.39 respectively. Myeik Shopping Mall recorded 0.267 hours (16.02 minutes), Market Garden recorded 0.417 hours (25.02 minutes) and Grade Jade is 0.4 hours (24 minutes) on the time a customer spent waiting for service. According to the data, Market Garden and Grade Jade are less effective average time spent on the system. Finally, equation (1) indicates that using input utilization was very high.

The least utilization was 0.78 (or 78%) at Grand Jade. The Myeik Shopping Mall was 0.80 (or 80%) and Market Garden was recorded 0.83 (or 83%) usage at the top of the list. This means servers primafacie. These supermarkets have been under stress. Once they start work, they rarely have time to rest that day. This is defined as the percentage of capacity used other than human resources. Existing physical building are under extreme pressure and they are under pressure maintenance and replacement costs.

4. CONCLUSION

The application of the queuing theory of supermarket in Myeik is discussed. Queuing theory is a simple and very practical theory of real life problems. In queuing theory is a model constructed so that queue lengths and waiting times can be predicted. Compared to supermarkets, the waiting time in queue at Myeik Shopping Mall's cashier is only 0.263 per hour, which is good for all services. The second is the Grand Jade, which is 0.4 hours waiting time and Market Garden is the least available service at 0.417 per hour. Queuing theory can be managed by reducing customer waiting time, thus saving time and reducing cashier's working hours, which can provide excellent benefits. This paper examines effective queue management of supermarkets in Myeik. Information on supermarkets in arrival rate, service rate, waiting time in queue and probability of potential customer to balk are provided.This paper shows a need of a numerical model for the supermarket management to understand the situation better. As a result of this paper, the popular queuing show problems for supermarkets have been created similar challenges may apply to many cities in developing countries.

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Lexical Relations in the Selected Proses by Bertrand Russell

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ABSTRACT: In learning the literature of a language, it is important for the learners to get a complete message when appreciating it. When presenting the literature, the writers' choice of words and the lexical relations of the words play an important role in depicting its essence. The present study endeavors how the words are lexically related in the proses. The aim of the study is to explore lexical relations in the selected proses. The objectives are to investigate what types of lexical relations are dominant in the selected proses and to examine the effects of using lexical relations in the selected proses. To fulfill the objectives of the study, two proses, How to Grow Old and A Free Man's Worship by Bertrand Russell are selected to analyze the data and the checklist of lexical relations stated by Saeed (2003) is used as an analytical tool. In the present study, only three types of lexical relations such as synonymy, antonymy and hyponymy are studied in the selected essays. It is found that antonymy is the dominant lexical relation in the selected proses. It can be said that the effective use of lexical relations in the proses improves the comprehensive skill of the readers and makes the readers get the contextualized meanings of the words in the selected proses. It can be said that the present research will make the learners more interested and active in learning literature and linguistics, especially semantics.

KEYWORDS: Lexical relation, synonymy, antonymy, hyponym

1. INTRODUCTION

Semantics is the study of the meanings of words and sentences (Saeeed, 2003: Pg 1). Meaning of a sentence depends on meaning of its words and how the words are structurally and semantically combined. One semantic approach concerned with the meanings of the words and how the meanings of the words are related to others is lexical relation. The present study mainly emphasizes on lexical relations in the prose.

Prose is a literary work that exhibits a natural flow of speech and grammatical structure. Prose writers always make use of literary techniques and choice of words to present a message to the readers. Bertrand Russell, a British philosopher and writer, creates his prose with the best choice of words to make the readers get the message presented in his prose, effectively and clearly. Moreover, the use of lexical relations is found in his prose. In the present study, his proses are selected to analyze lexical relations by using the theory of lexical relations proposed by John I. Saeed (2003).

1.1. LITERATURE REVIEW

In the present study, lexical relations in the selected proses are analyzed and classified. How to Grow Old and A Free Man's Worship by Bertrand Russell are selected to collect the data. Bertrand Russell (1872-1970) was a British philosopher and writer and he was born in Mouthmonshire into one of the most prominent aristocratic families in the United Kingdom.

1.2. RELATED RESEARCH

In the present study, the research conducted by Suwandi(2011) is used as a reference. Suwandi (2011) conducted a research entitled A semantic analysis of meaning properties and lexical relation in Essay writing of the fourth semester students of 2010/2011 academic year at FKIP UIR. The objectives of the research are (1) to find out the types of meaning properties and lexical relation in Essay writing of the fourth semester students of the English Department of FKIP and (2) to find out which types of meaning properties and lexical relation are dominant in Essay writing of the fourth semester students of the English Department at FKIP. It was found that 24% of ambiguity, 67.3% of redundancy, 8.6% of contradictory, 68.7% of hyponym, 5.2% of synonym and 26.3% of antonym in the Essay writing of the fourth semester students of the English Department at FKIP. It was also found that the dominant meaning properties and lexical relation are redundancy and hyponym in their writing.

2. THEORETICAL BACKGROUND

In the present research, the theory of lexical relations stated by John I. Saeed (2003) is used to investigate lexical relations found in the selected proses and key terms such as synonymy, antonymy and hyponymy are analyzed. The definitions for key terms are as follows:

Synonymy
Synonyms are different phonological words which have the same or very similar meanings.
Antonymy
Antonyms are words which are opposite in meaning.
Hyponymy
Hyponym is a relation of inclusion. A hyponym includes the meaning of a more general word. The more general term is called the superordinate or hypernym.

3. RESEARCH METHODOLOGY

The present study emphasizes on lexical relations in the selected proses by Bertrand Russell. The theory stated by Saeed (2003) is used to classify only three types of lexical relations such as synonym, antonym and hyponym.

3.1. DATA COLLECTION

The data are collected from the two selected proses by Bertrand Russell. The use of lexical relations in the

selected proses is collected and analyzed. The following tables and figures show synonyms, antonyms and hyponyms found in the selected proses. The samples of collecting data are presented.

Table 1: Synonyms in How to Grow Old

Sr. No	Sentence No.	Sentences
1	5	I can only **discover** one who did not live to a great age, and he died of a disease …..
	31	….. but human beings, owing to the length of infancy, **find** this difficult.
2	6	….. and her last day remained a **terror** to all her descendants.
	37	Some old people are oppressed by the **fear** of death.
3	10	She inquired the cause of his **melancholy** and he said that he had just parted
	23	….. in regrets for the good old days, or in **sadness** about friends who are dead.

Table 2: Synonyms in A Free Man's Worship

Sr. No	Sentence No.	Sentences
1	2	Had he not given them endless **joy**?
	91	To abandon the struggle for private **happiness**, to expel all the eagerness of temporary desire.
2	32	Such is the attitude inculcated in God's answer to Job out of the whirlwind: the **divine** power and knowledge are paraded.
	99	When their good and their evil have become **eternal** by the immorality of the past,…
3	31	Some, though they feel the demands of the ideal, will still consciously **reject** them,
	50	To defy with Promethean constancy a hostile universe, to keep its evil always in view, always activated hated, to **refuse** no pain…

Table 3: Antonyms in *How to Grow Old*

Sr. No.	Sentence No.	Sentences
1	1	In spite of this title, this article will really be on how not to grow **old**, which,….
	3	Although both my parents died **young**, I have done well…..
2	5	Of remoter ancestors I can only discover one who did not **live** to a great age,…..
	46	I should wish to **die** while still at work, knowing that others will carry on,…..
3	5	….. I can only discover one who did not live to a **great** age, and he died of a disease which is not rare,…..
	42	An individual human existence should be like a river—**small** at first,…..

Table 4: Antonyms in *A Free Man's Worship*

Sr. No.	Sentence No.	Sentences
1	15	And he gave God thanks for the strength that enabled him to forgo even the joys that were **possible**.
	56	It must be admitted that, of the things we desire, some, though they prove **impossible**, are yet real goods:…
2	15	And he gave God thanks for the strength that enabled him to forgo even the **joys** that were possible.
	98	But let us think only of their need—of the **sorrows**, the difficulties, perhaps the blindnesses,…
3	99	We were ready with encouragement, with sympathy, with **brave** words in which high courage glowed.
	101	…disdaining the **coward** terrors of the slave of Fate, to worship at the shrine that his own hands have built;..

Hyponyms in *How to Grow Old*

Table 5: Hyponymy relations of "time"

Sr. No.	Sentence No.	Original Sentence
1	1	In spite of this title, this article will really be on how not to grow old, which, at my **time** of life, is a much more important subject.
	14	…so she habitually spent the hours from **midnight** to **3:00 A.M.** in reading popular science.

The words "midnight" and "3:00 A.M." have a hyponymy relation. They are related to the meaning of "time". They are hyponyms of "time", the hypernym. The following tree-diagram shows hyponymy relations of "time".

Figure 1: The hierarchical relationship of "time"

Table 6: Hyponymy relations of "number"

Sr. No.	Sentence No.	Original Sentence
6	17	… statistical fact of the **number** of years you have already lived, still less of the probable brevity of your future.
	4	… at the age of **sixty-seven**, but my other **three** grandparents all lived to be over **eighty**.
	7	My maternal grandmother, after having **nine** children who survived, **one** who..
	10	…he said that he had just parted from his **two** grandchildren.

The words "sixty-seven", "three", "eighty", "ninety-two", "nine", "one", "two" and "seventy-two" have a hyponymy relation. They are related to the meaning of "number". They are hyponyms of "number", the hypernym. The following tree-diagram shows hyponymy relations of "number".
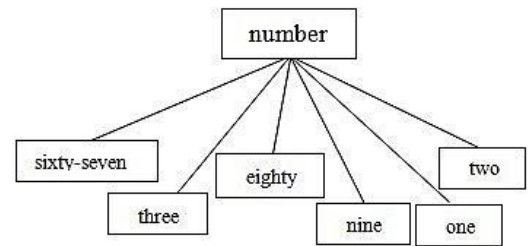
Hyponyms in *A Free Man's Worship*

Table 7: Hyponymy relations of "human"

Sr. No.	Sentence No.	Original Sentence
4	11	And Man stood aside from the struggle, resolving that God intended harmony to come out of chaos by **human** efforts.
	16	And God smiled; and when he saw that **Man** had become perfect in renunciation and worship,….
	23	… with the capacity of judging all the works of his unthinking **Mother.**

The words "Man" and "Mother" have a hyponymy relation. They are related to the meaning of "human". They are hyponyms of "human", the hypernym. The following tree-diagram shows hyponymy relations of "human".

Figure 3: The hierarchical relationship of "human"

Table 8: Hyponymy relations of "day"

Sr. No,	Sentence No.	Original Sentence
5	99	And so, when their **day** is over, when their good and their evil have become eternal….
	101	…for Man, condemned **today** to lose his dearest, **tomorrow** himself to pass through the gate of darkness.

The words "today" and "tomorrow" have a hyponymy relation. They are related to the meaning of "day". They are hyponyms of "day", the hypernym. The following tree-diagram shows hyponymy relations of "day".
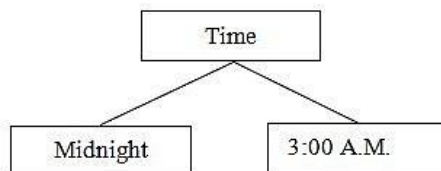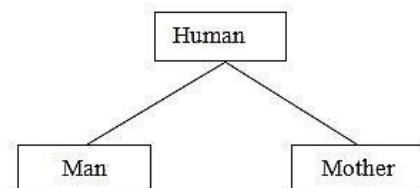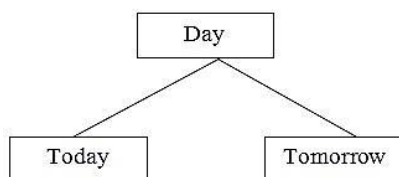

Figure 4: The hierarchical relationship of "day"

3.2. DATA ANALYSIS

Lexical relations found in the proses *How to Grow Old* and *A Free Man's Worship* by Bertrand Russell are classified and analyzed by using the theory of Saeed (2003).The use of synonymy, antonymy and hyponymy is analyzed. The frequencies and percentage of using lexical relations in two selected proses are shown in the following table. In the following table, Roman numerals I and II are used for the two selected proses.

Table 9: Lexical relations found in the selected proses by Bertrand Russell

Sr. No.	Lexical Relations	Freuen-cies		Total	Percent-age
		I	II		
1	Synonym	5	17	22	36.07%
2	Antonym	9	21	30	49.18%
3	Hyponym	6	3	9	14.75%
Total				61	100%

In the present study, 31.1% synonymy, 51.1% antonymy and 30% hyponymy are observed and it is found that antonymy is the most dominant lexical relations in the two selected proses by Bertrand Russell.

3.3. DATA INTERPRETATION

The frequencies of the lexical relations in the proses, *How to Grow Old* and *A Free Man's Worship* are shown in the following figure.
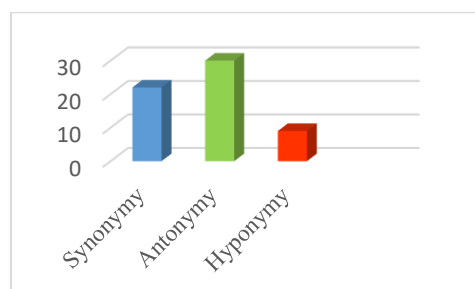
Figure 5: The frequencies of lexical relations in the two selected proses

4. FINDINGS AND DISCUSSION

In the present study, the data in the selected proses are analyzed from the semantic point of view.

The use of lexical relations in the selected proses makes the readers understand the message presented in the proses by the writer clearly and the readers can get the contextualized meaning of the proses.

To fulfill the objectives of the present study, it is found that the dominant lexical relation is antonymy and it can be said that Bertrand Russell can create his prose with the best choice of words and use of lexical relations. Moreover, he can present the same message by using different words effectively. The use of lexical relations make the readers know the contextualized meanings of the words and it improves the readers' better understanding and their reading skill. Furthermore, it can be said that the present research will be effective for ESL learners in studying linguistics, especially semantics and the learners become more interested and active in learning literature.

5. CONCLUSIONS

The lexical relations used in the selected proses are studied at the semantic level. The proses, *How to Grow Old and A Free Man's Worship* are selected to collect data. The use of synonyms, antonyms and hyponyms is analyzed and counted into percentage. It is found that the dominant lexical relation is antonymy and it can be concluded that using lexical relations help the readers to reveal the underlined meanings in the prose and they can get the message clearly and effectively. The present researcher would like to suggest that the analysis of using lexical relations can be explored in other literary works by using the theories by other scholars. Moreover, other kinds of lexical relations can be investigated to arouse the readers' interest in the writer's choice of words and to make the readers know the semantic level of words to get a complete message of the text.

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DISCOURSE MARKERS IN THE ESSAY DIARIES OF THE UNDERGRADUATE STUDENTS

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ABSTRACT: This study endeavors to examine the use of discourse markers in a small corpus writing of Second Year International Relations students from Yadanabon University. The aim of the research is to analyze the different types of discourse markers found in their essay diaries. Discourse markers are classified in this research from a semantic perspective, analyzing the misuse, overuse, underuse and inappropriate use of discourse markers in their writing. Fraser's (2005) classification of discourse markers is used as the working theory for the purpose of data collection and data analysis. The findings point out that nearly half of all the discourse markers used in the essays were misused or used in inappropriate way. It can be said that some students do not have enough understanding of how discourse markers function to shape better interpretation of a text and thus contribute to cohesion and coherence of the discourse. Considering the results of the research, it can be concluded that sufficient use of discourse markers in a piece of writing yields a better writing and greater understanding of the text. It is hoped that the present study can contribute to teaching and learning process in a way that empowers teachers to take discourse markers into account when they teach writing and that can raise students' awareness of how to use discourse markers efficiently to better the quality of their writings.

KEYWORDS: discourse marker, essay diaries, semantic perspective, cohesion, coherence

1. INTRODUTION

In language teaching, identifying the letters comes in first when we develop writing skills and then we gradually achieve other complex skills. Among four language skills, writing is the most intricate and difficult to master because it involves linguistic competence such as morphology and syntax. (Sharndama and Yakubu, 2013)

As Feng (2010) states, a good writing has to be both grammatical and cohesive. A smooth flow of ideas in the clauses and sentences gives great help in presenting a logical and well-organized text. The use of text-forming devices called discourse markers glues together discourse segments with the preceding ones to enhance the information the writer wants to deliver in a text. Discourse markers have a great influence on the cohesion and coherence of a piece of writing or a spoken discourse. They serve as a bridge to make separate messages in a discourse stick together, indicating the connection between them. So these markers should be sufficiently used both in spoken and written discourse to enable the readers and listeners to follow the growth of ideas in the discourse.

1.1 The Aim and Objective of the Study

The aim of this research is to analyze how undergraduate students used discourse markers to create semantic relationship in their essay diaries and the objective is to analyze the overuse, underuse and inappropriate use of DMs in them.

2. THEORETICAL BACKGROUND

2.1 Discourse Markers

Discourse means pieces of language larger than a sentence, functioning together to carry an idea or information. The linguistic devices that speakers or writers use to link language expressions together are called discourse markers. They are used to convey ideas, information and instructions in both conversation and written discourse.

Fraser's Taxonomy of Discourse Markers

Fraser (2005) identifies four discourse markers from a semantic standpoint which are described below with examples.

2.1.1 Contrastive Markers (CDMs)

Contrastive markers signal that the utterance following is either a rejection or a contrast with the preceding discourse. Expression of contrastive markers includes:

but, alternatively, although, contrariwise, contrary to expectations, conversely, despite (this/that), even so, however, in spite of (this/that), in comparison (with this/that), in contrast (to this/that), instead (of this/that), nevertheless, nonetheless, (this/that point) notwithstanding, on the other hand, on the contrary, rather (than this/that), regardless (of this/that), still, though, whereas, yet

2.1.2 Elaborative Markers (EDMs)

Elaborative markers signal that the utterance following includes some addition or clarification to the preceding discourse. Examples of elaborative markers are:

and, above all, also, alternatively, analogously, besides, by the same token, correspondingly, equally, for example, for instance, further(more), in addition, in

other words, in particular, likewise, more accurately, more importantly, more precisely, more to the point, moreover, on that basis, on top of it all, or, otherwise, rather, similarly, that is (to say)

2.1.3 Inferential Markers (IDMs)

Inferential markers signal a conclusion which follows from the preceding discourse and some examples are:

so, after all, all things considered, as a conclusion, as a consequence (of this/that), as a result (of this/that), because (of this/that), consequently, for this/that reason, hence, it follows that, accordingly, in this/that/any case, on this/that condition, on these/those grounds, then, therefore, thus

2.1.4 Temporal Markers (TDMs)

Temporal markers show a temporal relation in two actions taking place at the same time, or two events happening one after another. The class of temporal markers includes:

then, after, as soon as, before, eventually, finally, first, immediately afterwards, meantime, meanwhile, originally, second, subsequently, when

2.2 Related Research

In "Discourse Markers in English Writing", Feng (2010) investigates thirty-eight students who major in tourism management and English. The students were given a writing assignment to write an article titled "How Do You Think of English" without word limitation. 38 articles were collected and examined in harmony with Halliday and Hasan (1976) to find the presence and extent of discourse markers. Results showed that some students rarely use discourse markers to avoid unexpected mistakes. It was found that they employed as few discourse markers in their articles as possible and that was what made their writing quite odd. Moreover, the research also proves that most students did use a particular number of discourse markers in their essays but in incorrect or inappropriate ways. Thus, the researcher makes a conclusion highlighting the fact that discourse markers should be taken into account during teaching writing classes.

3. RESEARCH METHODOLOGY

3.1 Participants

50 second year students majoring in International Relations from Yadanabon University (2018-2019 Academic Year) contributed to this research.

3.2 Method

The students were asked to write an essay diary under the topic of "2017 was an amazing/terrible year" without word limitation. Students were asked to include in their essay diaries what they learnt over the past year, what they wanted to accomplish in life both in the short and long terms, what problems they wanted to fix and how they felt about what they did over the past year. Since the writing assignment is a kind of narrative plus descriptive writing, students were expected to apply all types of discourse markers to best express their

attitudes, events, ideas, opinions and situations when recounting their stories.

Students were assigned to do the writing task as a classroom activity and they had no idea that their writings would be examined. Students were asked to write freely and that since their essays would not be graded, they did not need to worry about grammar and spelling. Therefore, it can be assured that the students used discourse markers as they normally do. A small corpus of 50 writings was collected and a list of discourse markers identified in their essays was made and then examined how they used discourse markers to create semantic relations in their writings and if there would be inappropriate use of DMs in them.

4. DATA COLLECTION AND ANALYSIS

The following table reveals the use of discourse markers in students' essay diaries.

Table 1: Discourse Markers Used in the Students' Essay Diaries

Student No.	DMs Used	No. of Correct DMs	No. of Incorrect DMs
1	although, but, and2, because	3	2
2	but, and, because, because of	3	1
3	but2, and2, if, so2	3	4
4	and3, also, now	3	2
5	but3, and4, also, because2, so2	5	7
6	and2, because, so, after, at first2, now	4	4
7	although, and, when	3	-
8	and, so, when	3	-
9	but3, and, as, because, then2, when	5	4
10	and3, when4	4	3
11	and2, because, so, when	4	1
12	but, and, because, so, so that, now	3	3
13	and, by the way, because, so, firstly, later	2	4
14	and9, also, so	4	7
15	but, and2	3	-
16	if	1	-
17	but, and, because, later, now, when	4	2
18	also, because of, so	3	-
19	but, so3, since	3	2
20	still, because2	3	-
21	and2, so	3	-
22	and2, so2	3	1

Student No.	DMs Used	No. of Correct DMs	No. of Incorrect DMs
23	but, because of, when	2	2
24	but, and2, because2, because of, so, when2	6	3
25	but2, and4, because2, so, next	4	6
26	but, and4, also, if	5	2
27	because, if, when	2	1
28	although, but, and2, also, because2, when2, now	6	4
29	but, and2, because of, when	5	-
30	and3, also, so2, therefore, before2, now, since	6	5
31	and3, then, finally	2	3
32	and2, therefore, next	2	2
33	although, and, moreover, when	2	2
34	but, and3, besides, because of, first	4	3
35	and3, so, later,	3	2
36	But, and, because, from then on, now	2	3
37	but, however, besides, for example, after that, first	3	3
38	but, and4, so2, sometimes, besides, now	6	4
39	so2, because, when, next, nowadays	3	3
40	but, and2, because, for example, so, finally	3	4
41	but, however, besides, first, second, for example, after this, in the end	4	4
42	but2, so2, besides	3	2
43	but5, however, and, now2	5	4
44	But, besides, for example	3	-

Student No.	DMs Used	No. of Correct DMs	No. of Incorrect DMs
45	but, however, and, for, now2, firstly, secondly	5	3
46	and2, but2, so, then	5	1
47	but3, and3, for example, so, then2, besides, after all	7	5
48	on the other hand, even, and, so, now	3	2
49	but, and, because, from then on, now	3	2
50	but, on the other hand, and4, because, then, later	5	4
Total No. of DMs		181	126
		307	
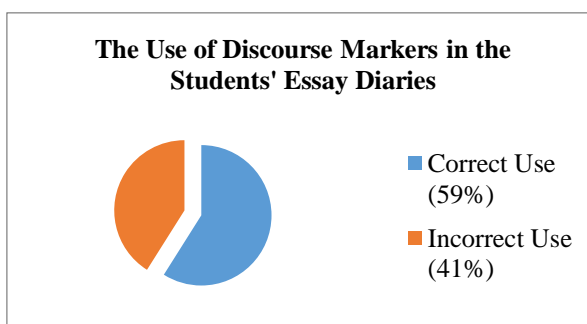
Figure 1: The Use of Discourse Markers in the Students' Essay Diaries

When teaching students how to write in English, some teachers put the focus on spelling mistakes, some emphasize on grammar, some concentrate on writing styles and some on structure. But discourse markers have been overlooked in writing classes. So, most students intuitively apply discourse markers just where and how they want them in their writings. Although a limited number of discourse markers are present in their writings, they do not really know how to use them correctly and properly. So, some mistakes or inappropriateness occur in their writings as shown in some extracts below.

E.g. 1. ___Although___ *our teacher taught us well* ___and___ *I had many friends, I learnt not only about IR but also others.* (Student 1)

Here the use of "*although*" is inappropriate semantically because there is no antithesis in these sentences.

E.g. 2. *I was an ugly and fat girl.* (___So___) *I lacked confidence about my appearance.* ___And___ *I* ___also___ *forgot to read books.* (___Moreover___), *I gave much*

attention to people who made me feel sadness. (Student 4)

Here the student tried to use discourse markers but she omitted two markers in the brackets. If the discourse markers in the brackets were added to the sentences, it would be more natural and fluent.

E.g. 3. *I have got many experiences about my life **because** I am standing on the world without bringing support from my parents, sometimes I was crying the whole night **because** I was **still** remembering them. **(But now)** The time is over, **and** **also** I can't study a little more **and** I thought I can't live without them.* (Student 5)

If discourse markers in the bracket *(But now)* were added to the sentence, it would give a better understanding of the context. The last line in this extract includes the inappropriate use of elaborative discourse markers, "*and*" and "*also*". The student is trying to add more details to his writing by the use of DMs but there seems to have no connection within these three sentences due to the incorrect use of DMs.

E.g. 4. ***At first**, I was scared of studying in Mandalay. **(But later)** I more realized about people **and** met both good and bad persons. **At first**, I didn't know how to treat people, especially my friends. I didn't know their mind. **(But)** after a few months, I knew more about my friends **and** knew how to build our relationship. **(Besides)** studying the IR lessons changed my habits and mind to treat other people. **(So)** I think my decision was right to attend IR subject.* (Student 6)

Student 6 avoided using as many discourse markers in his essay as possible. But this makes his writing a bit strange. There are only 4 discourse markers ("*and*" and "*at first*"). It would be better if the discourse markers in the brackets were added to the sentences.

E.g. 5. *I have many lessons in 2017 **because** I've never left my parents' heart.* (Student 12)

It is not right to use "*because*" to connect these two sentences. It seems that "*because*" in this context does not introduce a logical relation which represents cause-and-result relationship.

E.g. 6. *…… **And** I am interested in business. **So** I did a small online shopping in my Facebook. I **also** made a little friendship with the people. **And** I made hobbies to be happy in my leisure time. 2017 was a good year **and** this year gave me a lot of experiences such as selling things, social with people **and** so on….. **And** I **also** learnt about how to treat people and social world. **And** I had a lesson from 2017. That is "Don't waste time. Nobody can make time for you **and** respect your time"….. I studied **and** learnt many things were associated with life. **And***

*2017 gave me a lot of happiness….. **And** I fell in love with a girl older than me.* (Student 14)

The discourse marker "*and*" occurs too recurrently and without need. The student should also use other EDMs to express his ideas or add more information to his essay because the repeated use of a certain discourse marker makes the writing uninteresting and dull.

E.g. 7. *I was **still** a student over the last year. For me, 2017 is the most terrible year in my life. **Because** I met what I had never been in my life.* (Student 20)

"*Still*" is normally used to contrast the ideas in the previous sentence. Obviously, the use of "*still*" in this sentence is semantically inappropriate because it was used in the very first sentence of the essay and there is no preceding sentence to contrast the idea with.

E.g. 8. *I'm afraid **because of** I didn't know about university.* (Student 23)

Here the student confuses between "*because*" and "*because of*".

E.g. 9. *I started to learn international relations subject. It is a good major subject **but** I don't interest it. I don't interest it **but** I try to understand it. **When** looking back on 2017, I think it is a terrible year **because** I attended university with major which I dislike and uninterested. **But** I try the best to like it. **Then** I can understand it **and** I like it little. **As** my mother said, 'Wherever you are, you should try the best yourself. **Then** you can get a position". It is a lesson I learnt in 2017.* (Student 9)

Described above is from Student 9 who in his little piece of writing used all four types of discourse markers by whose ability it turns out to be a reader-friendly essay diary despite some grammatical mistakes.

E.g. 10. *I did nothing special things last year. **When** I attended in first year, I missed my school friends. **But** I'd got new friends. **And** I **also** found my soul mate. My family faced with hard times. That made me stressed for a long time. **When** looking back on 2017, I see myself as a good girl **because** I never gave my life up **although** there were many problems. **Now** I know how to communicate with new friends. **And** I know that I should study regularly even for the tutorial tests. It was an amazing year **because** I felt luckier than this year.* (Student 28)

Also, Student 28 applied all types of discourse marker in his/her diary. So it turned out to be well-constructed and of good-quality.

5. DISCUSSION AND CONCLUSION

The most dominant type of discourse marker is elaborative discourse markers followed by inferential discourse markers. This study suggests that discourse

markers are fairly presented in their writings in spite of their unequal distribution. Moreover, findings indicated that 126 out of 307 discourse markers used in all the students' essay diaries are found to have been misused or used in inappropriate way (See Table 1). That is nearly half of all the markers used in all the essays (See Figure 1). As explained in the extracts above, it can be guaranteed that abuse or inappropriate use of discourse markers creates confusion and thus leads to misinterpretation and misunderstanding in decoding the text. So readers may find it hard to infer a connection among the sentences or make sense out of what the writer is trying to deliver in his message. Considering the results of the research, it is found that discourse markers take an important stand in cohesion and coherence of a text. When it comes to creating cohesion and coherence in a text, in addition to text-forming devices such as reference, substitution, ellipsis, reiteration and collocation, there is also a great possibility of using discourse markers in any kind of writing. Therefore, teachers should know how students use discourse markers and whether they use them in their writing pieces correctly and properly in order to have a good preparation when they plan to teach writing.

The use of discourse markers indicates the quality of a text which evokes the readers' personal feelings or opinions. The employment of discourse markers makes the writing more interesting and expressive when recounting their real-life situations, life, love, loss, ups and downs. Accordingly, it is found that discourse markers contribute to logical relations and semantic interpretation of a text.

The results of the study would be of great support for the teacher to decide the inadequate use of discourse markers by the students. Teachers will be able to plan and prepare their teaching based on the strengths and weaknesses of the students. As it is clear from the findings, it can be said that most students do not have enough understanding of how discourse markers function to shape better interpretation of a discourse. It is important for the students to know the multiplicity of discourse markers suitable for what they want to convey in their essays when adding more information, contrasting ideas, emphasizing, showing cause-effect relations and chronological order.

It is also noted that there is overuse of some discourse makers. Recurrence of a particular discourse marker usually sounds dull and uninteresting and is a clear sign of the writer's little awareness of discourse markers.

This paper discusses the use of DMs and their effects on the meaning of the writings in second year IR students' writings from the semantic point of view. In order to make their English writing more cohesive and more coherent, students are very likely to use discourse markers in their essays. So when we teach writing, discourse markers should be taken into consideration in order that students would be aware of how the sufficient use of discourse markers in a piece of writing can help construct the text logically and make the connections between sentences more apparent.

In conclusion, the present research throws light on the overuse, underuse and misuse of discourse markers as an aspect of achieving semantic relations in the students' essays. The present study can possibly help teachers and learners of English become more aware of cohesion and use it properly to attain the preferred effects in their writings. The effective use of discourse markers shows the greater language mastery with stronger relations. So the knowledge of using discourse markers when producing a text is seen to be a cause for successful writing and of maturity in thought. It is hoped that the present study can give an insight into the effective use of discourse markers in accordance with their linguistic features and in a way will help the student writers avoid making the same inappropriate use of discourse markers in a written text.

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DEVELOPING THE WRITING SKILL OF THE ENGINEERING STUDENTS AT THE TECHNOLOGICAL UNIVERSITY (HMAWBI)

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ABSTRACT: Writing is an essential feature of communication for the engineering students. So, the English teachers have become increasingly concerned with the need to train the students of science and technology in writing. Writing is one of the most important and most frequently used medium of communication. This paper aims to improve the writing skill of the engineering students at technological universities. To achieve this aim, the needs survey was done through questionnaires with the help of the students. The result showed that most of the engineering students are weak in English writing skill because they are lack of concentration and lack of practice. In this paper, the strategies which inspire the students' interest and motivation in teaching and learning are described. The approaches to tackling writing tasks are suggested, depending on their needs, lacks and wants which have been defined in the survey. As a result, this paper is hoped to be of some help in developing the writing skill of the students.

KEYWORDS: *strategies; communication; writing skill; developing*

1. INTRODUCTION

Writing is not a natural activity. All physically and mentally normal people learn to speak a language. Yet every student has to be taught how to write in English. Unlike speaking, writing is almost always learned in school. Moreover, it is the language skill which is difficult to acquire. The most important job of a teacher is to teach the learners so that they can learn useful things as the best learning environment in order to get a balance of learning opportunities.

Writing is always a part of the syllabus in teaching English. However, it can be used for a variety of purposes, ranging from being merely a 'backup' for teaching grammar to a major syllabus strand in its own right, where the ability to have effective writing is seen as a key objective to proficiency for learners. When writing is done in practice, students frequently have more time to think than they do in oral activities. They can go through what they know in their minds, and even search for what they want to know in dictionaries, grammar books, or other reference material to help them.

Writing encourages students to focus on the accurate language use and, because they think as they write, it may well provoke language development as

they resolve problems which the writing puts into their minds. It is not surprising, then, that learners face with many difficulties in mastering writing skills in English. In this paper, the writer describes some kinds of writing techniques which are essential for engineering students.

2. LITERATURE REVIEW

The writer should consider a number of features in writing. Because writing is used for a wide variety of purposes, it is produced in many different forms. In the first place, the writer clearly has an audience in mind of their writing. In the second place, the writer has clearly changed their mind on more than one occasion, both deleting and adding items on the list. Lastly, it is worth nothing the use of a foreign word, obviously known to the writer. In this part, four kinds of writing which are necessary for engineering students will be presented.

2.1 Creative Writing

The term "creative writing" suggests imaginative tasks such as writing poetry, stories and play. Such activities have a number of features to recommend them. Chief amongst these is that the end result is often felt to be some kind of achievement, and that "most people feel pride in their work and want it to be read". This is significantly more marked for creative writing than for other more standard written products.

Creative writing is "a journey of self-discovery, and self-discovery promotes effective learning". When teachers set up imaginative writing tasks so that their students thoroughly engaged, those students frequently strive harder than usual to produce a greater variety correct and appropriate than they might for more routine assignments. While students are writing a simple poem about someone they care about, or while they are trying to construct a narrative or tell stories of their childhood, for example, they are tapping into their own experiences. This provides powerful motivation to find the right words to express such experience. Creative writing also provokes the kind of input – output relationship.

In order to bolster the "product pride" that students may feel when they have written creatively, it is necessary to provider an appropriate audience. Apart from teachers, the whole class can also be such an audience. Teachers can put students' writing up on a class notice board, or copy it and include it in class magazine. Teachers can make anthologies and distribute them to friends, parents, and other teachers.

There is always a danger that students may find writing imaginatively difficult. Having "nothing to say"

they may find creative writing a painful and de-motivating experience associated in their minds with a sense of frustration and failure. A lot will depend upon how teachers encourage students. It is also important not to expect whole composition from the very first. Teachers need to build up creative writing bit by bit, starting with phrases and sentences before expecting whole conversation.

2.2 Essay Writing

An essay is a piece of writing which is written to a set of writing conventions. An essay can have many purposes, but the basic structure is the same no matter what. The art of good essay writing lies in having something interesting to say and saying it clearly, concisely. Writing an essay can be an arduous and time-consuming task to tackle, but when students are able to write essays with confidence, other types of assessment will seem relatively easy. If they have not had much experience with essay writing in the past, sitting down to write the first one at college or university can seem a daunting experience, but there are really only five basic stages in producing a good essay: analyse the essay title; find and organise the relevant material, making appropriate notes; make a plan; produce a rough draft; and write the essay.

2.3 Report Writing

A report is the formal writing up for a piece of research or project work. Reports are usually written in a concise style, giving precise detail. Report writing will involve using skills of application – applying what has been learned to a particular situation, rather than investigating and discussing theory. The secret of successful report writing is to approach it systematically and present material in a simple, clear and logical way. The ability to write effective reports is an important and valuable skill for all students to acquire. The preparation for a report is divided logically into four stages – preliminary, preparation, writing and review.

2.4 Letter Writing

If students are taking an English language examination, there is a strong chance that they will have to face with questions involving letter-writing in its various forms. Whether they have qualified or not in the field of examination-taking, they will be confronted with letter-writing for the rest of their lives. In fact, any business needs people who can read and write letters effectively.

There are three main kinds of letters: formal letters; informal letters; business letters. A formal letter has some of the qualities of a business letter. Many of the letters that people have to write can be described as 'business or formal' letters. They are letters to business firms written by private individuals. Another kind of formal letter involves correspondence with people who have an official position. Students themselves may be writing to someone in an official capacity of their own or simply as private individuals.

Informal letters are letters to friends and relatives. Business-like qualities are not the first consideration in such letters, which are much more 'free and easy' than formal letters. Hitting the right note is the first consideration in informal letter-writing.

When students are practising letter-writing, they can study carefully and use it as a working model.

3. RESEARCH METHOD

This research has been done by using qualitative method. It is used to gain an understanding of reasons, opinions, and motivations. It also provides insights into the problem and helps to develop ideas for the engineering students. It was done in 2018-2019 academic year.

3.1 Participants

There are 150 engineering students who participated in this study and they were third year students from the Technological University (Hmawbi).

3.2. Need Analysis and Questionnaire

The purpose of this questionnaire is to determine the English language needs of the students. Please fill it in as completely as possible. Thank you very much for your cooperation.

1. What is your purpose(s) of learning English?
(a) To master the English language ☐
(b) To pass the examination only ☐
(c) To deal with the major subjects because they are taught in English ☐
(d) To be familiar with an international language ☐

2. What types of writing do you have to learn?
(a) Essay ☐
(b) Letter ☐
(c) Report ☐
(d) Summary ☐
Other (if any) --
----

3. What are the main problems you face in writing English?
(a) Grammar ☐
(b) Punctuation ☐
(c) Spelling ☐
(d) Others ☐
(Please specify) --
----

4. Do you think the writing skills are necessary in your career?
Yes ☐   No ☐

5. If so, to what extent?
(a) To great extent ☐
(b) To some extent ☐
(c) To little extent ☐

6. How often do you have to do writing exercises in your calss?
(a) Very often ☐
(b) Sometimes ☐
(c) Often ☐
(d) Never ☐

7. How do you find writing tasks you are doing?
(a) Boring ☐

(b) Interesting ☐
(c) Easy ☐
(d) Difficult ☐
8. How important are the writing skills in your class?
(a) Critically important ☐
(b) Very important ☐
(c) Somewhat important ☐
(d) Minimally important ☐
9. Is there enough practice for developing writing skills in the classroom?
Yes ☐   No ☐
10. How efficient are you in writing skills?
(a) Excellent ☐
(b) Fair ☐
(c) Good ☐
(d) Poor ☐

3.3 Data-Analysis and Results

Although data were collected from 150 engineering students within three days, only 120 students responded to this. According to the data collected in the questionnaires, in question number one, two and three, 90% of students said that the purpose of learning English is to be familiar with an international language; the types of writing they have to learn are essay, letter and report writing and the main problem they faced in writing English is grammar. In question number four and five, 85% of students answered that writing skills are necessary in their career. In question number six, seven and eight, 80% of students said that they often have to do writing exercises in their class. The writing skills are very important and they are very interested in writing exercises. In question number nine, 75% of students said that they have enough practice for developing writing skills in the classroom. In the last question, 70% of students answered that their writing skill is fair.

According to the students' responses to the questionnaire, some are interested in learning English writing skills such as report writing, formal letters, informal letters, and business letters. Some students answered that they are taught English from the kindergarten to university level and they really have certain exposure to the language to some extent. However, they still find it difficult to write in English. Thus, it can be concluded that they lack practices even though they have over ten years of learning English. Most of the students think that they should improve four integrated skills – reading, writing, speaking, and listening – but they do not know that developing writing skill is really important and is a must for engineering students. Although they are the university students, they cannot construct their own sentences. They are not able to choose correct usage. Most of the students are unable to express their ideas in writing and they are weak in producing grammatically correct sentences. They need much more practices and to know writing strategies.

So, the following strategies were introduced in the classroom. Then the students were asked to write essay, letter, report etc. as classroom activities in the class. As a result, the author has found the improvement of students' writing skills.

4. STRATEGIES FOR DEVELOPING WRITING SKILL

To develop their writing skill, this research paper will discuss and suggest some kinds of writing strategies.

4.1 Planning

The experienced writers plan what they are going to write. Before starting to write or type, they try to know and decide what it is they are going to say. For some writers, this may involve making detailed notes. For others, a few jotted words may be enough. Still others may not actually write down any preliminary notes at all since they may do all their planning in their heads. But they will have planned; nevertheless, the writer has thought about what is needed before writing on the piece of paper.

When planning, writers have to think about three main issues. In the first place, they have to consider the purpose of their writing since this will influence not only on the type of the text they wish to produce, but also the language they use, and the information they choose to include. Secondly, the experienced writers think of the audience they are writing for, since this will influence not only on the shape of the writing, but also the choice of language – whether, for example, it is formal or informal in tone. Thirdly, the writers have to consider the content structure of the piece – that is, how well to sequence the facts, ideas, or arguments which they have decided to include.

The first version of a piece of writing can be referred as a draft. This first 'go' at a text is often done on the assumption that it will be amended later. As the writing process proceeds into editing, a number of drafts may be produced on the way to the final version.

4.2 Editing

Once writers have produced a draft, they, then, usually read through what they have written to see where it works and where it does not. Perhaps, the order of the information is not clear. Perhaps the way something is written is ambiguous or confusing. They may then move paragraphs around or write a new introduction. They may use a different form of words for a particular sentence. More skilled writers tend to look at issues of general meaning and overall structure before concentrating on detailed features such as individual words and grammatical accuracy. The latter two are important and are often dealt with the later in the process.

Reflecting and revising are often helped by other readers who comment and make suggestions. Another reader's reaction to a piece of writing will help the author to make appropriate revisions.

4.3 Final Version

Once writers have edited their draft, making the changes, they consider to be necessary, they produce their final version. This may look considerably different from both the original plan and the first draft, because

the things have changed in the editing process. But the writer is now ready to send the written text to his intended audience. These stages can be represented in the following way:

planning ➡ drafting ➡ editing ➡ final draft

However, there are two reasons why this diagram is not entirely satisfactory. In the first place, it tells little about how much weight is given to each stage, but, more important by suggesting that the process of writing is linear it misrepresents the way in which the majority of writers produce written text.

There are three types of writing for learning; reinforcement writing, preparation for writing and activity writing (Harmer 2005).

4.3.1 Reinforcement Writing

Writing has always been used as a means of reinforcing language that has been taught. In its simplest form, teachers often ask students to write sentences using recently learnt grammar. And then, they might be given the instructions. Reinforcement writing, however, need not be confined to sentence writing. Students can also be asked to write the paragraphs or longer compositions to practise certain recently focused-on aspects of language or paragraph and text construction. Students might be asked to write a story about something that happened to them as a good way of having them practise past tenses. They could be asked to write a description of someone they know because this is a good way of getting them to use the character and physical description vocabularies they have been studying. Clearly, the aim of these activities is to give students opportunities to remember 'new' language better. Just the act of writing sentences makes them think about the new grammar or vocabularies in a more considered way than if the teacher asked them to provide instant spoken examples.

4.3.2 Preparation for Writing

Writing is frequently useful as preparation for some other activity, in particular when students write sentences as a preamble to discussion activities. This gives students time to think up ideas rather than having to come up with instant fluent opinions, something that many, especially at lower levels, find difficult and awkward. Students may be asked to write a sentence, saying what their opinion is about a certain topic. Another technique, when a discussion topic is given to a class, is for students to talk in groups to prepare their arguments. They can make written notes which they may use later during the discussion phase. In these cases, where writing has been used as preparation for something else, it is an immensely enabling skill even though it is not the main focus of an activity.

4.3.3 Activity Writing

Writing can also be used as an integral part of a larger activity where the focus is on something such as language practice, acting out, or speaking. Teachers often ask students to write short dialogues which they will then act out. The dialogues are often most useful if planned to practise particular functional areas, such as inviting or suggesting. Students work in pairs to make the dialogue and, where possible, the teacher goes to help them as they write. They now have something they can read out or act out in the class.

Writing is also used in questionnaire-type activities. The teacher then asks them all to stand up and circulate around the class asking their colleagues the questions they have previously prepared. They write down the answers and then report back to the class on what they have found out. Once again, writing is used to help students perform a different kind of activity. Students need to be able to write to do these activities, but the activities do not teach students to write.

5. CONCLUSIONS

Students on the writing skill may have very specific needs or this skill may simply aim to develop their writing proficiency. The content for writing activities will rely on the type of writing students are learning. Some approaches to teaching writing composition are explained to be able to write the composition correctly without wasting time. Teachers have many different roles in writing class, depending on the level of the class, the type of writing the students are learning and the stage in the writing process the students are engaged in. The essential types of writing should be taught as a model of learning not only a means to remediate deficiencies in writing skill.

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A STUDY ON THREE TYPES OF DEIXIS FOUND IN THE SHORT STORY *TWO THANKSGIVING DAY GENTLEMEN* BY O. HENRY

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ABSTRACT*:* The present paper mainly focuses on a study of three types of deixis found in the short story *Two Thanksgiving Day Gentlemen* by O. Henry. The aim of the study is to explore the use of deictic expressions found in this short story. The objectives are to identify, classify and analyze deictic expressions used in the short story and to study which one is more favoured in this text. This research takes in the form of quantitative method. The data were counted, analyzed and interpreted by using Levinson's (1983) theory. According to the findings, the most dominant type is personal deixis in this story. The results proved that different kinds of deixis make the story more interesting and enable the readers to understand the message of the story conveyed by the author.

KEYWORDS: *deixis, person deixis, temporal deixis, spatial deixis*

1. INTRODUCTION

There are two reasons why the present research is conducted. Firstly, it is assumed that analyzing deixis in O.Henry's *Two Thanksgiving Day Gentlemen* can describe how important deictic expressions in interpreting the text are. Secondly, such a detailed study of uses of deixis can convey the relationship between the structure of languages and the contexts in which they are used. This study is helpful to deepen our comprehension concerns with deixis and is beneficial to enhance our understanding of four skills in the second language.

2. THEORETICAL BACKGROUND

The term "deixis" was defined by different scholars in different ways. Lyons (1977:377) defines deixis the location and identification of persons, objects, events, processes and activities talked about, or referred to, in relation to the spatial-temporal context created and sustained by the act of utterance and the participation in it, typically, of a single speaker and at least one addressee (as cited in Andreea Stapleton).

Moreover, Levinson (1983) states that deixis is within the domain of pragmatics because it directly concerns the relationship between the structure of language and the context in which they are used. In other words, words and phrases cannot be understood without the contextual information.

Thus, it is a crucial subject in the field of pragmatics. Three deictic categories are identified in the literature. These are: **person deixis** (*I, we, you*), **spatial deixis** (*this, that, here, there, under*), **temporal deixis** (*now, then, today, on Friday*). Levinson (1983) also demonstrates that a word or phrase which points out the different meaning the words have in various situations is called deictic expression. Deictic expressions can be mostly found in face-to-face spoken interaction.

Levinson (1983) notes that there are five types of deixis: person deixis, spatial deixis, temporal deixis, discourse deixis and social deixis. Among them, three types of deixis are conducted in this research.

2.1 Person deixis

The term of person deixis was defined by Levinson (1983) as grammatical marker of participant role to the speech event. The category of first person means the speaker's reference to himself, the second person is to encode the speaker's reference to one or more addressees and third person refers to person and entities which are neither speakers nor addressees of the utterance in question. All of these categories have singular or plural pronoun. He also states that person deixis clearly operates on basic three part divisions exemplified by the pronouns for first person (*I*), second person (*you*) and third person (*he, she, it*). On the other hand, definite and specific pronouns like *this woman, that present, these books*, etc. are also deictic expressions because they require situational information for the listener to make out the referents or the meanings. Different scholars define the term 'person deixis' in different ways.

2.2. Spatial Deixis

Levinson (1983) points out that place deixis focuses on the specification of locations relative to the location of the participants in the speech event. Spatial deixis is relative to the speaker's location. Those are proximal (or near the speaker) and distal (or non-proximal, sometimes close to addressee and away from speaker). Then spatial deixis can be known not only from the use of demonstrative pronouns such as "this" and "that" but also from the demonstrative adverb of place such as "here" and "there".

2.3. Temporal Deixis

Levinson (1983) defines temporal deixis as an expression in relation to point to certain period when the utterances are produced by the speaker. These language resources are the adjectives of time in the line yesterday, now and tomorrow, and the verb tenses. The verb sometimes also has another function besides referring to a specific time. For example:

A: I live here now

B: I live there then

The present tense is the proximal form as in (a) and the past tense is distal form as in (b). The deictic items use reference that can only be determined in relation to the time of the utterance in which they occur. Such as:

This, last, next day, week, month, year.

Now, then, ago, later, soon, before

Yesterday, today, tomorrow.

3. RESEARCH METHODOLOGY

This research uses descriptive research. The descriptive research conducts to describe information of the real condition and situation. In this chapter, data collection and data analysis, and data interpretation are described.

3.1. Data collection and data analysis of person deixis

Table 1 shows the types of person deixis in the short story *Two Thanksgiving Day Gentlemen* by O. Henry.

Table 1 Kinds of person deixis in the short story *Two Thanksgiving Day Gentlemen* by O. Henry

No.	Person Deixis		Quantity	Percentages
1	Singular first person	I	4	3%
2	Singular first person	me	2	1%
3	Singular first person	my	3	3%
4	Plural first person	we	3	3%
5	Plural first person	us	1	1%
6	Plural first person	our	1	1%
7	Singular second person	you	4	3%
8	Singular second person	you	5	4%
9	Singular second person	your	2	1%
10	Plural third person	they	16	14%
11	Plural third person	them	3	3%
12	Plural third person	themselves	1	1%
13	Singular third	he	38	32%

No.				
	person			
14	singular third person	him	17	6%
15	Singular third person	his	19	16%
16	Singular third person	it	6	5%
17	Singular third person	it	3	3%
	Total		118	100%

3.2 Data collection and data analysis of spatial deixis

And the types of spatial deixis in the short story *Two Thanksgiving Day Gentlemen* by O. Henry are described in table 2.

Table 2 Kinds of spatial deixis in the short story *Two Thanksgiving Day Gentlemen* by O. Henry

No.	Spatial Deixis	Quantity	Percentages
1	here	4	10%
2	in	10	25%
3	on	6	15%
4	there	5	12.5%
5	from	1	2.5%
6	in the front of	1	2.5%
7	near	2	5%
8	up	1	2.5%
9	upon	1	2.5%
10	away	1	2.5%
11	outside	1	2.5%
12	comes	1	2.5%
13	to	1	2.5%
14	at the door	1	2.5%
15	went south	1	2.5%
16	went north	1	2.5%
17	around the first corner	1	2.5%
18	over	1	2.5%
	Total	40	100%

3.3 Data collection and data analysis of temporal deixis

Then table 3 indicates the types of temporal deixis in the short story *Two Thanksgiving Day Gentlemen* by O. Henry.

Table 3 Kinds of temporal deixis in the short story *Two Thanksgiving Day Gentlemen* by O. Henry

No.	Temporal Deixis	Quantity	Percentages
1	Every….	8	17%
2	on the first Thanksgiving Day	1	2.17%
3	now	1	2.17%
4	on thanksgiving Day	4	9%
5	today	1	2.17%
6	at noon	1	2.17%
7	always	3	7%
8	before	3	7%
9	ten minutes	1	2.17%

10	then	6	13%
11	coming across	1	2.17%
12	toward	1	2.17%
13	last year	1	2.17%
14	the year before	1	2.17%
15	on this day of Thanksgiving	1	2.17%
16	In the winter	1	2.17%
17	In the spring	1	2.17%
18	In the summer	1	2.17%
19	In the fall season	1	2.17%
20	some day	1	2.17%
21	and then	1	2.17%
22	in an hour	1	2.17%
22	one minute	1	2.17%
22	an hour later	1	2.17%
22	after	1	2.17%
22	three days	1	2.17%
22	in the afternoon	1	2.17%
	Total	46	100%

3.4 Data interpretation

In dealing with data interpretation, some sentences which used some deictic expressions from table 1, table 2 and table 3 are randomly selected and interpreted.

1.On Thanksgiving Day at noon **they** always sent a servant to stand at the door.

There is one deixis in this sentence. It is the word "they". The word "they" can be replaced by "two old ladies". The type of the deixis "they" is the plural third person deixis. The function of the deixis is the subject of the sentence.

2. For that blessing **you** and **I** can give thanks on this day of thanksgiving.

There are two deictic expressions: "you" and "I". The word "you" can be replaced by non-deictic referent "Stuffy Pete". The type of the deixis "you" is the singular second person deixis. The function of the deixis is the subject of the sentence. Then the word "I" can be replaced by "the Old Gentleman". The type of the deixis "I" is the singular third person deixis. The function of the deixis is the subject of the sentence.

3. "Thank **you** for **my** Thanksgiving dinner."

The words "you" and "my" are deixis in this sentence. The word "you" can be replaced by non-deictic referent "the Old Gentleman ". The type of the deixis "you" is the singular second person deixis. The function of the deixis is the object of the sentence. Then the word "my" can be replaced by non-deictic referent "Stuffy Pete's". The type of the deixis "my" is the singular first person deixis. The function of the deixis is the possession of the sentence.

4. He continued to eat in order to keep it **there**.

The word "there" is deictic expression in this sentence. The word "there" refers to the place, on the Old Gentleman's face. The type of the deixis "there" is spatial deixis. The function of that deixis is the distal form.

5. They were known **here**.

In this sentence, the word "here" is deixis. The word "here" refers to the restaurant. The type of the deixis

"here" is spatial deixis. The function of that deixis is the proximal form.

6. Then he stood up heavily and started to go **to** the wrong door.

This sentence consists of one deictic expression. It is the word "to". The word "to" means the location which shows Stuffy Pete started to go. The type of the deixis "to" is spatial deixis. The function of that deixis is the distal form.

7. His hair was whiter and thinner than it had been **last year.**

There is one deictic expression in this sentence. It is the phrase "last year". The phrase "last year" means the year before. The type of the deixis "last year" is temporal deixis. The function of that deixis is to refer to the time at which his hair was whiter and thinner than it had been the year before.

8. **In the winter** he grew a few flowers there.

This sentence involves one deictic expression. It is the phrase "in the winter". The phrase "in the winter" refers to the cold season at which the Old Gentleman planted a few flowers. The type of the deixis "in the winter" is the temporal deixis. The function of that deixis is the distal form.

9. But **today** Pete was not hungry.

This sentence involves one deictic expression: "today". The word "today" indicates to present time that the speaker utters the utterance. The type of the deixis "today" is temporal deixis. The function of that deixis is to show the present condition.

4. FINDINGS AND DISCUSSION

The major 89 sentences found in the story *Two Thanksgiving Day Gentlemen* were randomly selected and sorted out into three types; person deixis, spatial deixis and temporal deixis based on the theory of Levinson (1983).

Table 4: Frequency and Percentage of deixis in the short story *Two Thanksgiving Day Gentlemen* by O. Henry

Types of Deixis	Frequency	Percentage
Person deixis	118	58%
Spatial deixis	40	20%
Temporal deixis	46	22%
Total	204	100%

According to Table 4, the largest number of deixis is person deictic at 118 (58%), followed by temporal at 46 (22%), and the least number of spatial deixis is at 40 (20%). Due to the frequent use of person deixis, the readers are able to know the roles of participants in the speech event. And spatial deixis can make the readers to know the distance of the speech situation between the participants happened. Finally, temporal deixis serves as a mean of creating atmosphere so that the readers are able to visualize the picturesque of the story in their eyes and establish an appropriate state of mind for events to come.

5. CONCLUSION

Through the use of deixis, misunderstanding will not happen because both the speaker and the addressee will understand what is talking about. The frequent use of person deixis in the story enables the readers to understand the personal traits of the characters in the story and to make an effect on his story to be read easily without repeating the name of the characters again and again. Then the use of spatial deixis enables the readers to know the specification of locations relative to points of reference in the speech event. Finally, the use of temporal deixis shows the position of the referent of actions and events in time. The readers can infer meaning from the contextual clues not only without looking up the dictionary but also by analyzing the different types of deixis in the short story and they can comprehend the story vividly through the use of different types of deixis. Moreover, the context of the instructions, the status of those involved, the intention of the speaker, the place and time of these instructions in teaching, learning and workplaces can be clearly understand by comprehending the use of deixis.

Keys to Activities

1. The President gives **it** to **us every year**.
2. **They** were the Puritans.
3. **They** were some people who landed **on our** Atlantic shore.
4. But those people ate a large bird called turkey **on the first Thanksgiving Day**.
5. And **now here** is the story to prove to **you** that **we** have old traditions **in** this new country.
6. **They** are growing older more quickly than traditions **in** old countries.
7. **We** do everything quickly.
8. Stuffy Pete sat down **on a seat in** the New York City park named Union Square.
9. **Every Thanksgiving Day for nine years he** had sat **down there** at one **in the afternoon**.
10. **Every time**, things had happened to **him**.
11. **They** were wonderful things.
12. **On those other Thanksgiving Days he** had been hungry.
13. But many of **them** seem to think that the poor are hungry only **on Thanksgiving Day**.
14. But **today** Pete was not hungry.

15. **He** had come **from** a dinner so big that **he** had almost no power to move.
16. **His** breath was short.
17. **His** body had suddenly become too big for **his** clothes; **it** seemed ready to break out of **them**.
18. **They** were torn.
19. **You** could see **his** skin through a hole **in the front of his** shirt.
20. **He** had been passing a large house **near** the beginning of that great broad street called Fifth Avenue.
21. **It** was the home of two old ladies of an old family.
22. There were certain things **they** always did.
23. **On Thanksgiving Day at noon they always** sent a servant to stand at the door.
24. There **he** waited for the first hungry person who walked by.
25. Stuffy Pete happened to pass by on **his** way **to** the park**.**
26. The servant had gathered **him** in.
27. Stuffy Pete sat **in** the park looking straight **before him** for **ten minutes**.
28. **Then he** felt a desire to look in another direction.
29. **Then his** eyes grew wider and **his** breath stopped.
30. For the Old Gentleman was **coming across** Fourth Avenue **toward** Stuffy's seat.
31. **Every Thanksgiving Day for nine years** the Old Gentleman had come **there** to find Stuffy Pete **on his seat**.
32. **Every Thanksgiving Day for nine years he** had found Stuffy **there**.
33. **Then he** had led Stuffy **to** a restaurant and watched **him** eat a big dinner.
34. **They** do these things more easily **in** old countries like England.
35. But **in** this young country, **we** must think about **them**.
36. The Old Gentleman loved **his** country.
37. **He** believed **he** was helping to build a great American tradition.
38. Nine years is a long time **here**.
39. **He** wore eye-glasses.
40. **His** hair was whiter and thinner than **it** had been **last year**.
41. **His** leg did not seem as strong as **they** had seemed **the year before**.
42. But **he** could not move from **his** seat.
43. For that blessing **you** and **I** can give thanks **on this day of thanksgiving.**
44. If **you** will come with **me**, **my** man, **I** will give **you** a dinner that will surely make **your** body feels as thankful as **your** mind."
45. That is what the Old Gentleman said **every time**.
46. The words **themselves** were almost a tradition.

47. But now **he** looked **up** at the Old Gentleman's face with tears of suffering **in** his eyes.
48. The snow turned quickly to water when **it** fell **upon his** hot face.
49. **He** turned **away**, with **his** back to the wind, and **he** did not see Stuffy's eyes.
50. Stuffy had **always** wondered why the Old Gentleman seemed sad as **he** spoke.
51. **He** did not know that **it** was because the Old Gentleman was wishing that **he** had a son.
52. A son would stand proud and strong **before** Stuffy, and say: "In remembrance of **my** father."
53. **Then it** would really be a tradition.
54. **He** lived **in** a room **in** one of the old houses **near** the park.
55. **In the winter he** grew a few flowers **there**.
56. **In the spring he** walked **on** Fifth Avenue.
57. **In the summer he** lived **in** a farmhouse **in** the hills **outside** New York, and **he** talked of a strange bug **he** hopped **some day** to find.
58. **In the fall season he** gave Stuffy a dinner.
59. **His** face was getting older **every year**, but **his** clothes were very clean and fresh.
60. **And then** Stuffy made a strange noise.
61. **He** was trying to speak.
62. As the Old Gentleman had heard the noise nine times **before**, **he** understood **it**.
63. **He** knew that Stuffy was accepting.
64. "Thank **you**, **I**'m very hungry."
65. **His** desire for food **on Thanksgiving Day** was not **his** own.
66. The Old Gentleman led Stuffy **to** the restaurant and **to** the same table where **they** had **always** gone.
67. T**hey** were known **here**.
68. "**Here comes** that old man," said a waiter, "that buys that old no-good fellow a dinner **every Thanksgiving**."
69. Stuffy saw the look of happiness **on the Old Gentleman's face**.
70. **He** continued to eat in order to keep **it there**.
71. **In an hour** the battle was finished.
72. "Thank **you**," Stuffy said. "
73. Thank **you** for **my** Thanksgiving dinner."
74. **Then he** stood up heavily and started to go **to** the wrong door.
75. **They** said goodbye, as **they** did each year, **at the door**.
76. The Old Gentleman **went south**, and Stuffy **went north**.
77. Stuffy went **around the first corner**, and stood for **one minute**.
78. **Then he** fell.
79. **There he** was found.
80. **He** was picked up and taken **to** a hospital.

81. **They** put **him on a bed**, and began to try to discover what strange sickness had made **him** fall.
82. **And an hour later** the Old Gentleman was brought **to** the same hospital.
83. And **they** put **him on another bed**, and began to try to discover what **his** sickness could be.
84. **After** a little time one of the doctors met another doctor, and **they** talked.
85. "That nice old gentleman **over there**," **he** said.
86. "Do **you** know what's wrong with **him**?
87. **He** is almost dead for need of food.
88. A very proud old man, **I** think.
89. **He** told **me he** has had nothing to eat for **three days**."

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ENHANCING STUDENTS' VOCABULARY DEVELOPMENT THROUGH EFFECTIVE TEACHING AND LEARNING STRATEGIES

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ABSTRACT: Vocabulary plays a crucial role in English Foreign Language (EFL) acquisition, particularly for university students. Vocabulary acquisition is the process of learning new words in the language that involves reading, writing, listening, and speaking skills. If students acquire deep and rich vocabulary knowledge, they will convey their message more efficiently. This paper aims to develop students' vocabulary growth through effective teaching, and learning strategies. It surveyed 52 students studying EFL and 16 English language teachers. The data collected by questionnaires were analyzed quantitatively and qualitatively. According to the survey questionnaires, the results express various attitudes of the participants towards vocabulary learning and teaching. Students are weak and less practice in vocabulary. Students also claim to learn most vocabulary activities for vocabulary development. The understanding of students' perceptions of vocabulary learning enables teachers' design teaching activities that enhance students' lexical competence. Understanding vocabulary nature is vital to the method of selecting appropriate strategies that enable students to master the vocabulary they need to learn. Teaching and applying a good range of vocabulary learning strategies will help university students to bear in mind the importance of vocabulary learning and improve their vocabulary development.

KEYWORDS: *strategies, appropriate, teaching and learning*, *EFL, vocabulary development.*

1. INTRODUCTION

Vocabulary knowledge plays a significant role in overall academic success. Vocabulary refers to a set of lexemes, including single words, compound words, and idioms (Venkateswaran, 1998).

An important objective of teaching a language is to develop the vocabulary of the learners and help them to comprehend the meaning of words and sentences. One of the main purposes of this study is to support teachers in doing something to promote their students' vocabulary development. Another purpose in promoting students' vocabulary growth/development is to make students use effective vocabulary learning strategies and to make them better able to understand what they read. Vocabulary is an essential part of English Foreign Language teaching and learning.

Shastri (2010) expressed that vocabulary forms the central core of language learning. It is needed words to express thoughts and ideas to point objects, and to describe things. Lack of vocabulary is a major handicap for the learners of English as a Second/Foreign Language.

Watkins (2005) stated that even at higher levels, successful learning of new words and phrases is often a way by which students can see that they are making progress. So, an approach that values the ability to communicate, vocabulary is essential, and in recent years vocabulary has become increasingly central to language teaching. Takač (2008) revealed that *Vocabulary learning strategies* are activities, behaviors, techniques used by learners to facilitate vocabulary learning. Vocabulary learning strategies can help learners to discover lexical items, and to internalize, store, retrieve, and actively use these in language production. *Vocabulary teaching strategies* refer to everything teachers do or should do to help learners learn the vocabulary of the target language.

The research questions for this study are:
1. What vocabulary teaching strategies are important in teaching vocabulary?
2. What vocabulary learning strategies are essential in learning vocabulary?

The results of this study could be useful for English language teachers as well as English language learners.

2. LITERATURE REVIEW

2.1 The Importance of Vocabulary

Vocabulary acquisition is the largest and most important task facing language learners. Thornbury (2002) said that "Without grammar very little can be conveyed, without vocabulary nothing can be conveyed" The learner can say very little with grammar, but she/he can say almost anything with words.

Vocabulary is closely associated with intelligence and knowledge. A person who knows more words can speak and think more precisely about the world than a person who has poor words. A person who knows the terms *scarlet*, *crimson*, *azure*, and *indigo* can think about colors in a different way than a person who doesn't. A person who knows about *balks*, *bunts,* the *double switch* can think about baseball in a different way than a person who doesn't. Words divide the world; the more words the learners have, the more complex ways they can think about the world.

For teachers, vocabulary is important because of the huge differences that exist among their students. There are dramatic differences in exposure to new words among families of different social classes. The

more words the learners know, the easier it is to learn more words.

2.2 Motivation and Vocabulary Growth

Motivation is an important role in vocabulary learning, as it does in any other kind of learning. In the field of motivation, a distinction made between 'intrinsic' and 'extrinsic' motivation.

Intrinsic motivation is when a person does something because he/she feels that doing it is intrinsically valuable – it's worth doing for its own sake. Extrinsic motivation is when a person does something not because he/she necessarily wants to do it, but instead for some reward. Studies of motivation generally show that intrinsic motivation promotes learning more effectively than extrinsic motivation.

A part of intrinsic motivation is a sense of the power of words. Understanding word choice is a powerful communicative tool. Although the teachers might need to tell this to students once in a while, it is much more important to show them. Later, the teachers give some more specific ideas about how to show students the power of words. However, the teachers want to go over a few more general principles relating to motivating students to learn words. Another important aspect of motivation is helping students to experience a reasonably high rate of success in instructional activities. If the teachers already know the word, they are likely to underestimate the difficulty of the task for students who do not already know it.

The main motivation for vocabulary learning must be intrinsic that students must see vocabulary as being something interesting and useful. The most important part of the motivation for vocabulary learning is helping students learn to value words.

3. EFFECTIVE STRATEGIES FOR STUDENTS' VOCABULARY DEVELOPMENT

According to the research question no.1, the following strategies express effectively in teaching vocabulary.

3.1 Direct Vocabulary Instruction

Teachers should provide direct instruction of some word meanings in addition to providing rich exposure to words. It offers a few guidelines and strategies that can be adapted to teach academic vocabulary (Vaughn, 2010).

Teach specific vocabulary words directly. There is no best approach to teaching academic vocabulary. The chosen method should reflect how extensively the teachers want their students to understand the vocabulary selected.

Teach important words in depth. Integrate the most important academic vocabulary into their instructional routine so that students encounter the words often and have many opportunities to apply their knowledge of words in reading, writing, and discussion. Focus on the relationship between selected words to be important concepts or themes.

When students read from a variety of text types, the teacher should heighten students' awareness of the language in the world around them, and develop

the skills of students to learn new vocabulary on their own. The suggestions are as follow:

Teach word-learning strategies. Teaching students how to use word parts and context to determine the meaning of unknown words has shown to benefit vocabulary learning. Many of the words in the resource lend themselves to word study activities such as teaching common root words, prefixes, and suffixes. Teaching students these word foundations can go way toward helping students understand related vocabulary words.

Foster a love of words and language in the classroom. If students bear in mind the vocabulary around them and actively engage in using new words and seeing the relationships among words, students will take ownership of their language learning.

Encourage wide reading. Students do not learn most of the words through direct instruction. There are too many unknown words in teaching through direct instruction. Incidental learning and exposure to new words increase in reading when students have many opportunities to read a variety of texts.

Use student-friendly definitions of words and concepts. Provide a student-friendly definition in their own words, what they think the word means. This activity will help the teachers monitor students' understanding and help them develop a better understanding of the word.

Use graphs, pictures, or symbols when appropriate. This approach is especially useful for mathematical concepts that are difficult to explain verbally, but that is well suited to nonverbal explanations.

Yopp. et al. (2009) revealed that some principles should be guided to teachers in order to teach word meanings. They are the following.

1. Instruction should provide more than definitional information about words. It must acknowledge the complexity of words and ensure that students give ample contextual information about words.
2. Instruction should provide repeated exposure to words understudy and multiple opportunities for students to use and practice the words.
3. Instruction should encourage students to think about relationships among word meanings.
4. Instruction should actively engage students in learning tasks.
5. A variety of instructional practices should be employed.
6.

3.2 Teaching Word-learning Strategies

An effective vocabulary teaching is incomplete if teachers do not develop students' word-learning strategies. The suggested strategies include using word parts and using dictionaries.

3.2.1 Using Word Parts

One strategy that supports students' abilities to acquire new words is the use of word parts to unlock meaning. Freeman, (2009) stated that words that make up the academic vocabulary of English often have Latin or Greek roots as well as prefixes and suffixes. These word parts can be used by students to infer the meaning of words in context. For example, the root *ject* comes from the Latin word *iacere* and the meaning is "to

throw". This root appears in words like *eject, reject, adjective, conjecture,* and *inject*.

Prefixes and Suffixes provide different information. Prefixes can change or add to the meaning of the root while suffixes signal the part of speech. For example, the word *re* means "again" or "back", so *replace* must mean something like "to place again" or "to put back". In a word *placement*, the suffix (*-ment*) doesn't add to or change the meaning of *place*. The use of prefixes and suffixes help to develop productivity in learners. The use of prefixes (*un-, dis-, mis-, ir-, il-, etc.*) forms the antonyms of the words such as *able – unable, match – mismatch, satisfy – dissatisfy, responsible – irresponsible, logical – illogical*. Suffixes help in changing the grammatical form of the words. For example:

The noun form verb: denial – deny, departure – depart.
                    entertainment – entertain.
The noun from the adjective: generosity –   generous, ability – able.
The adjective from the noun: bookish – book, misty – mist.
The verb form adjective: ripen – ripe, freshen – fresh.

Understanding the meaning of roots, prefixes, and suffixes will support students' ability to induce word meanings. If students know the word *uni-* means 'one', they will be able to conclude that a *unicycle* is a one-wheeled cycle. Understanding the prefix *uni-* will contribute to students' understanding of words such as *uniform, unicorn, unicellular*.

3.2.2 Using Dictionary

Another word-learning strategy is the effective use of a dictionary. A dictionary is a powerful tool, especially when students are motivated to understand the meaning of a word.

Freeman offered tips for helping students use dictionaries effectively and efficiently. The suggested guidelines are as follow:

1. When reading a definition, students should be sure to read all of it, not just part of it.
2. Students should remember that many words have more than one meaning.
3. Students should be sure to check all the definitions a dictionary gives for a word, not just one of them.
4. Students should decide which definition makes sense in the passage in which they found the word.
5. The dictionary often works best when students already have some ideas of a word's meaning. This makes the dictionary particularly useful for checking on a word that they want to use in their writing.

At a higher level, the learners can ask to prepare a list of the library books according to the title or the author's name following the alphabetical order. This will help them in preparing a bibliography. They should ask to collect other information in the dictionary such as American or British spelling, different meanings of the word, colloquial variety, and grammatical information about the word whether it is a noun (countable/uncountable), adjective or adverb, verb (transitive/intransitive), etc. They should be encouraged to refer to different dictionaries to get the desired information and enrich their knowledge.

These are some of the effective strategies that help learners to learn vocabulary development.

3.3 Teaching Vocabulary through Wide Reading

The teacher should emphasize on vocabulary expansion of the learners. This can achieve through *wide reading*.

Wide reading is the largest single source of vocabulary growth/development. Thus, encouraging wide reading is important for vocabulary learning. Wide reading is essential for increasing students' vocabulary size. It should notice that 25 minutes of reading a day could lead to a gain of over 2,000 words per year. Reading is the best method of increasing students' vocabulary.

Students of all ages learn words through reading. Several factors influence the process. These include the following:

1. *The difficulty of the text and the student's level of comprehension*: The text should match the student's level of comprehension. Students are less likely to learn new words from materials that they find difficult or easy.
2. *The word's conceptual difficulty*: The more difficult the word, the less likely a student will learn it from context. Less conceptually difficult words are easier to learn.
3. *How vital the word is to understanding*: The more important the word is to the ideas in the text, the more likely the student will learn it.
4. *The informativeness of the context*: The more information provided by the surrounding text, the more likely the student will learn the word.
5. *The number of times the word encounter*: Repeated encounters with the word increase the likelihood that the student will learn it.

These factors are important to remember that the teachers plan reading experiences for their students. The notion of wide reading is important. Although extensive reading on a single topic offers the depth, breadth is also crucial for expanding vocabulary.

4. DEVELOPING STUDENTS' VOCABULARY GROWTH THROUGH ACTIVITIES

Students benefit from working in small groups on tasks with cooperative effort. Here are some small-group activities that provide practice with prefixes, suffixes, and grammatical functions for vocabulary development.

Task 1:
Complete the needed word in the following sentences. The <u>underlined prefix</u> will tell you that which word is needed. (across, again, poorly, before, below)
1. A <u>sub</u>heading does not go above the main heading; it goes <u>below</u>.
2. A <u>trans</u>atlantic flight goes _____ the Atlantic.
3. A <u>pre</u>fix is a word-part that comes _____.
4. When we <u>re</u>write something, we write it _____.
5. <u>Mal</u>adjusted people are people who adjust _____.
Task 2:
Fill each blank with the appropriate noun form that corresponds to the <u>underlined adjectives</u> in the

following sentences. Then copy each completed sentence.

1. I don't know how <u>high</u> that ceiling is, but I'm going to measure its ____.
2. I know the river is <u>deep</u>, but I don't know its exact ____.
3. He is <u>proud</u>, and there are good reasons for his ____.
4. <u>Young</u> people seldom appreciate their ____.
5. Sometimes a <u>strong</u> man is not aware of his ____.

Task 3:
Indicate whether each word in the following list is a verb, a noun, an adjective, or adverb, and <u>underline</u> the part of the word which shows its grammatical functions.
Example: empha<u>size</u> (verb)

1. friendship
2. attractive
3. suggestion
4. punishment
5. sadness
6. finally
7. humorous
8. fortunately
9. possible
10. recognize
11. beautify

5. METHODOLOGY

This study aimed to find out the attitudes of both students and teachers as regards vocabulary learning and teaching. It also aimed to assist in promoting students' vocabulary development and some strategies for teaching and learning vocabulary.

5.1 Participants

The participants in the study served the purpose of the study. A total of 52 foreign language learners who were undergraduate students and 16 English language teachers participated in the study. All the participants are interested in the survey.

5.2 Instruments

The main instrument of this study was the questionnaires. The survey consisted of two questionnaires with the best choice question. There were four items in each questionnaire. The first questionnaire assessed the students' attitudes on vocabulary learning. The second questionnaire assessed the teachers' attitudes on teaching vocabulary.

5.3 Procedure

According to the standard procedure, the questionnaires were distributed to the participants. It was made clear to the participants that the survey was anonymous, and they should answer the questions honestly. The participants also invited to ask for help or additional explanations if necessary.

6. RESULTS

The results of the survey show in Table 1 and 2. The data collected were statistically analyzed. Table 1 shows the results of students' attitudes on vocabulary

learning. Table 2 shows the results of teachers' attitudes on vocabulary teaching.

Table1. Students' attitudes towards vocabulary learning

1. What's your attitude on the importance of learning English vocabulary?		
	Number of respondent	(%)
Very important	35	67.3
Important	17	32.7
Not important	0	0
2. In your opinion, what is the difficulty in your vocabulary learning?		
	Number of respondent	(%)
Pronunciation	6	11.5
Spelling	12	23.1
Meaning	23	44.2
Word collocations	11	21.2
3. Which area do you need to develop your vocabulary growth/development?		
	Number of respondent	(%)
Motivation	7	13.5
Lexical knowledge	11	21.2
Reading	12	23
Practicing vocabulary activities	22	42.3
4. Which area of knowledge have you learned is helpful for your vocabulary development?		
	Number of respondent	(%)
Contexts of word	19	36.5
Lexical knowledge	17	32.7
Word learning strategy	11	21.2
Western culture	5	9.6

According to the responses of the students' questionnaire, as shown in Table 1, the vast majority of the students (67.3 %) views on learning English vocabulary are very important. Many of the students have positive attitudes on the importance of having a large vocabulary. Most of the students (44.2 %) stated that they have difficulty in meaning. Other difficulties of the students are (11.5 %) for pronunciation, (23.1 %) for spelling, and (21.2 %) for word collocations in learning vocabulary. The highest (42.3 %) of students stated that they need to practice vocabulary activities for developing vocabulary growth. The rest respondents' opinions are (23 %) for reading, (13.5 %) for motivation and (21.2 %) for lexical knowledge in needing to develop vocabulary. The findings suggest that teachers of English could let their students spend more time practicing vocabulary activities. Another interesting finding, the highest (36.5 %) of students respond that the context of the word of knowledge they have learned is helpful for their vocabulary learning. (32.7 %) and (21.2%) of students revealed that lexical knowledge and word learning strategy that they have learned is respectively helpful for their vocabulary development. So, the results suggest that the teacher should train their students in doing lexical activities and

learning vocabulary strategies towards vocabulary development.

Table2. Teachers' attitudes towards vocabulary teaching

1. What's your attitude on the importance of teaching English vocabulary?		
	Number of respondent	(%)
Very important	10	62.5
Important	6	37.5
Not important	0	0
2. In your opinion, what is the difficulty in your vocabulary teaching?		
	Number of respondent	(%)
Pronunciation	2	12.5
Spelling	3	18.7
Meaning	7	43.8
Word collocations	4	25
3. What area do the students require to develop their vocabulary growth/development?		
	Number of respondent	(%)
Motivation	3	18.7
Lexical knowledge	2	12.5
Reading	5	31.3
Practicing vocabulary activities	6	37.5
4. Which area of knowledge have you learned is helpful for your vocabulary teaching?		
	Number of respondent	(%)
Contexts of word	5	31.3
Lexical knowledge	7	43.7
Word learning strategy	3	18.7
Western culture	1	6.3

According to the teachers' responses to the questionnaire, all the respondents state that they understand the importance of teaching vocabulary as it is the basis for understanding and using language. Different participants have different opinions on the difficulty in vocabulary teaching. The responses are (12.5 %) for pronunciation, (18.7 %) for spelling, (43.8 %) for meaning, and (25 %) for word collocations respectively. To increase the students' vocabulary development, the teachers' views on the students' requirements are (18.7% for motivation), (12.5% for lexical knowledge), (31.3%, reading), and (37.5%, practicing vocabulary activities). (43.7% and 31.3%) of the teachers responded that lexical knowledge and context of the word that they have learned is helpful for their vocabulary teaching. (18.7%) of respondents stated that word-learning strategies are helpful for their vocabulary teaching. The least (6.3%) expressed that western culture is helpful for vocabulary teaching.

Overall, the results state that all the respondents understand that lack of appropriate vocabulary affects their academic performance or expresses higher-level complex thoughts, opinions, or concepts and to comprehend academic texts. Students will need to control their vocabulary learning. It needs to introduce the students to a variety of strategies and let them decide which ones are right for them. The strategies described in this paper are suitable for both teachers and learners in teaching and learning vocabulary.

7. SUGGESTED VOCABULARY LEARNING STRATEGIES

The teachers can only a limited amount of vocabulary in the classroom; beyond that, the students need to learn on their own. The more initiative they take in identifying, noticing, learning, reviewing new vocabulary, the better they are. According to the research question no.2, here are some ideas the teacher can recommend to the students.

1. *Look for new words.* A student should keep an eye out for new words or expressions that he/she comes across in books, on the Internet, on notices or advertisements. He/she should make a mental note, and then look them up or ask someone what they mean at the first opportunity.

2. *Don't be afraid to ask.* If the student doesn't understand something, he/she should ask someone who knows the language better than he/she does. People are usually happy to help.

3. *Keep a vocabulary notebook (paper or digital).* The student should write new items as he/she learns them. He/she can tick them off or delete them when he/she is sure he/she knows them. He/she should keep the notebook in his/her pocket or bag so that when he/she comes across new items during the day, he/she can pull it out and jot down the new item immediately.

4. *Connect keywords with words in the student's language.* When the student learns a new word, he/she should try to find a word or name in his/her language that sounds similar and invent a reason to connect the two. It works as a mnemonic, helping him/her to recall the new word when he/she is needed.

5. *Spend ten minutes a day.* The student should take ten minutes every day to go over new vocabulary he/she has learned, whether in his/her notebook, on the card, or the textbook.

6. *Use bilingual dictionaries.* The student should use a bilingual rather than monolingual dictionary to look up a word for the first time: it's faster and easier to use, and just as accurate. He/she should use a monolingual dictionary for further explanations and examples.

7. *Use digital dictionaries.* The student should install an online dictionary on the "favorites/bookmarks" on the internet browser, or his/her mobile phone or tablet to look things up quickly and easily: The *Cambridge Dictionary Online*, for example (http://www.dictionary.cambridge.org/). There are also commercial programs that enable students to click on an unknown word and get a translation or dictionary definition.

8. *Use the dictionary while the student is reading (paper or digital).* The student should check out the meanings of occasional new words, and write them down. If he/she stops to look up every single word, he/she won't enjoy the reading or improve his/her reading fluency.

9.

8. CONCLUSIONS

English is an important language that everyone should learn it. Good English language users come with good vocabulary knowledge.

This study will convince the teachers, and learners that vocabulary instruction is important and strategies provided for enhancing the students' word knowledge. Word-learning strategies increase students' academic vocabulary. Teaching word parts can be a useful word-learning strategy if used judiciously. Teachers can show students how prefixes provide clues about the part of speech. By teaching word parts, teachers can instill an interest in words and help students develop academic language. The tasks should be learners' interests and needs. Reference skills with the help of a dictionary should be developed in the learners. They should be encouraged to look up a word in the dictionary. Teachers should monitor students using a dictionary appropriately. It noted that wide reading is the primary engine that drives vocabulary growth. It is needed to make an effort that the teachers have to know not only the importance of vocabulary knowledge to educational success, but also the complexity of the vocabulary problem.

The strategies expressed in the previous sections are suitable to help and improve EFL learners' vocabulary development. Learners of English as a Foreign Language should apply and implement the right vocabulary learning strategies, so that, they can study with ease and excel significantly. The further research question is, 'What is the learners' perception of Vocabulary Teaching Strategies (VTS) used by their teachers?'

Keys to Activities

Task 1:

6. A <u>sub</u>heading does not go above the main heading; it goes <u>below</u>.
1. A <u>trans</u>atlantic flight goes <u>across</u> the Atlantic.
2. A <u>pre</u>fix is a word-part that comes <u>before</u>.
3. When we <u>re</u>write something, we write it <u>again</u>.
4. <u>Mal</u>adjusted people are people who adjust <u>poorly</u>.

Task 2:

1. I don't know how <u>high</u> that ceiling is, but I'm going to measure its <u>height</u>.
2. I know the river is <u>deep</u>, but I don't know its exact <u>depth</u>.
3. He is <u>proud</u>, and there are good reasons for his <u>pride</u>.
4. <u>Young</u> people seldom appreciate their <u>youth</u>.
5. Sometimes a <u>strong</u> man is not aware of his <u>strength</u>.

Task 3:

1. friend<u>ship</u> (noun)
2. attract<u>ive</u> (adjective)
3. sugges<u>tion</u> (noun)
4. punish<u>ment</u> (noun)
5. sad<u>ness</u> (noun)
6. fina<u>lly</u> (adverb)
7. humor<u>ous</u> (adjective)
8. fortunate<u>ly</u> (adverb)
9. possi<u>ble</u> (adjective)
10. recog<u>nize</u> (verb)
11. beaut<u>ify</u> (verb)

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